Geometry-Teacher Notes

Overview	
Students will work with dimensions, area, and volume of pyramids and prisms as they investigate the mathematics behind the design of a Western Palins Indian Tipi.	 Prerequisite Understandings Area, volume, surface area Calculate slant heights of pyramids and prisms using the Pythagorean Theorem or trigonometry.

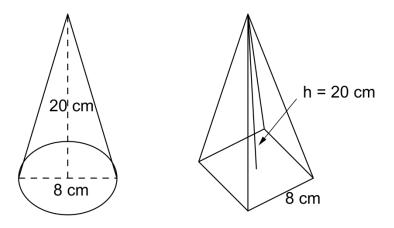
Curriculum Content	
Alaska Mathematics Content Standards	 G-MG.1. Use geometric shapes, their measures, and their properties to describe objects. G-MG.3. Apply geometric methods to solve design problems. G.GMD.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
Alaska Mathematical Practices	 4. <i>Model with mathematics</i>: Students create a polygonal pyramid model (or cone) for a tipi. 6. <i>Attend to precision</i>: Students' construction of the model tests precision in scaling down a real-life shape.

Task	
Supplies	Core Activity
 Stiff paper for making models (optional) 	The emphasis in this activity is leading students toward exploring their own thinking rather than the teacher leading them to any particular solution.
Launch	Extension(s)
Review and strengthen student skills on the Pythagorean Theorem, area formulas, volume formulas, and trigonometry with triangles.	Explore the traditional size of a tipi from research with old pictures, using people's heights as a determinate of the scale of a photo. Explore the usable floor area for standing or sitting in a tipi, find the maximum volume for a tipi with a given base and surface area, or compute the number of buffalo hides that were needed to cover a tipi.

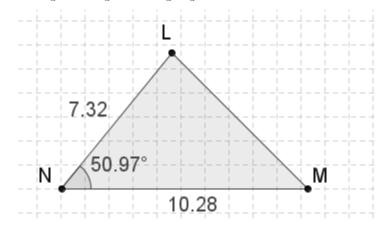
Launch

Use geometric and trigonometric formulas to analyze the following geometric shape.

1. Compare the slant height and the volume of a cone with a height of 20 cm and a diameter of the base of 8 cm to the slant height and the volume of a square pyramid with a height of 20 cm and a base diagonal of 8 cm.



2. Find the altitude, missing side length, missing angles' measurements, and area of the triangle below.



3. Describe how to find the surface area and volume of pyramids with a regular polygon as a base.

Activity

A strong part of western American culture is the Plains Indian Tipi. This structure could be as simple as a 3 pole structure (a tetrahedron shape) but usually was much more elaborate, with 12 or more poles giving the structures a cone shaped appearance.

Your task will be to create an engineering design and a **scale model of a life-sized tipi**. Provide specifications, and describe the processes you used to come up with the measurements. The life-sized tipi you will be modeling is a 12-pole tipi with a height of 18 feet to the cross pole, and has a perimeter of 48 feet (4 feet between poles).



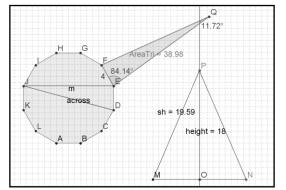
- 1. Prepare a detailed specification sheet. It should include the following:
 - A title page with your name.
 - A drawing of the area of the base of the tipi with measurements of lengths and areas.
 - A drawing of the net (the cover of the tipi) with area calculations.
 - The calculated volume of the tipi.
 - A supplemental page where you outline the formulas and calculations you did to find the information above.
- 2. Create a scale model of the tipi.
 - Make your model from any materials you wish.
 - Make a title card to place with the model that includes its title, your name, and the scale of the model.
- 3. Decorate the cover with some traditional Plains Indian designs or geometric patterns.

Results from the Classroom

This task was tackled by a class that had just started the year with trigonometry and had an easy fluency with formulas and angles.

Fewer than 20% of the students chose the option of using a computer program to build their models. One student's computer-generated scale model of a side of the Tipi (the base, one side, and a side view) is shown on the right.

Most students chose to make all drawings and all calculations by hand on cm grid paper. The actual scale models turned in by most students used a scale of 1 cm = 2 ft. It was clear that the size

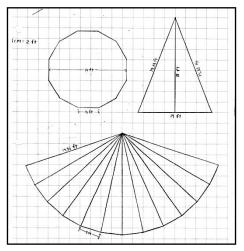


of the paper and open discussions during the project design and building phase resulted in students sharing their ideas for scale with each other. Most scale models came out very close in size because of this similarity of scale. To get more variety in the final results, have students use different scales or vary the heights and number of poles.

The project did result in some interesting errors by students. Take the list of formulas typed on one paper (see right). This student assumed the shape was just a cone and made some other basic errors such as the formula for circumference and the slant surface area (1/2 of a circle?). Cone: $V = (1/3)\pi r^{2}h$ Circle/Base: Perimeter= πd^{2} $A = \pi r^{2}$ Net: $A = \pi r^{2}/2$

height = 18ff	lin= 3ft height = 6 in
scale malle short heigh	t actual Spant height
10, 46.52	
(19.99)	(a tit
$6^{2} + 2.55^{2} = x^{2}$ 12.48 = x ²	$7.61^2 + 11^2 = x^2$
12.48 - x ² x= 6.52	382.36 · x2 x=19,55
Surface Area	Sustace trea
SA= TICL	SA . 16.30
SA= T.255.6.52 SA= 52.14	SA = 467.20

The student work to the far right made extensive use of the Pythagorean Theorem and did not rely on trigonometric calculations for most angles in the drawings.



Many of the drawings looked like the one on the left. They were accurate, neat, readable, and involved the correct scale. Labeling all angles in the drawings would help reinforce the applications of trigonometry to this project.

As a final note, the real tipis of the America Plains Indians were actually made of animal skins and were stitched together in patterns that could be adjusted as they were being built. Trial and error along with experience certainly has a value!

Geometry