High School Mathematical Content Standards

Courses and Transitions

The high school standards specify the mathematics that all students should study in order to be career and college ready. They are organized into **Conceptual Categories**, which are intended to portray a coherent view of high school mathematics. A student's work with any set of standards crosses a number of traditional course boundaries. For example, the Functions Standards would apply to different courses such as Algebra I or Algebra II.

These standards do not mandate the sequence of high school courses. However, the organization of high school courses is a critical component to implementation of the standards. It is a district decision how to design course offerings covering the mathematics standards. Districts can use the traditional approach of Algebra I, Geometry, and Algebra II or implement an integrated approach. There are various high school math pathways to be considered and likely additional model pathways based on these standards will become available as well.

The standards themselves do not dictate curriculum, pedagogy, or delivery of content. In particular, districts may handle the transition to high school in different ways. For example, many students in the U.S. today take Algebra I in the 8th grade, and in some districts and states this is a requirement. By completing grade 7 standards successfully, students have met the prerequisites and are prepared for Algebra I by 8th grade. The standards are designed to permit districts and states to continue existing policies concerning Algebra I in 8th grade.

[College-Ready:] Another major transition is the transition from high school to post-secondary education for college and careers. The evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by the words in brackets, [College-Ready:], in these standards. Indeed, some of the highest priority content for college and career readiness comes from grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume. Because important standards for college and career readiness are distributed across grades and courses, systems for evaluating college and career readiness should reach as far back in the standards as grades 6-8.

Narrative of Standards - Number and Quantity

Numbers and Number Systems - During the years from kindergarten to 8th grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3... . Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities - In real-world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Modeling Standards - Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by the words in brackets as [Specific Modeling Standards:].

Number and Quantity Standards

The Real Number System	N –RN

Extend the properties of exponents to rational exponents.

- N-RN.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*
- N-RN.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. *For example: Write equivalent representations that utilize both positive and negative exponents.*

Use properties of rational and irrational numbers.

- N-RN.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

[Specific Modeling Standards:]

Quantities	N – Q

Reason quantitatively and use units to solve problems.

- N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.
- N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

The Complex Number System	N - CN

Perform arithmetic operations with complex numbers.

- N-CN.1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
- N-CN.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- [College-Ready:] N-CN.3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

- [College-Ready:] N-CN.4. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
- [College-Ready:] N-CN.5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(1 - \sqrt{3}i)^3 = 8$ *because* $(1 - \sqrt{3}i)$ *has modulus 2 and argument 120°.*
- [College-Ready:] N-CN.6. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

- N-CN.7. Solve quadratic equations with real coefficients that have complex solutions.
- [College-Ready:] N-CN.8. Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.
- [College-Ready:] N-CN.9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities	N – VM

Represent and model with vector quantities.

- [College-Ready:] N-VM.1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $||\boldsymbol{v}||$, $\boldsymbol{v}$).
- [College-Ready:] N-VM.2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- [College-Ready:] N-VM.3. Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

- [College-Ready:] N-VM.4. Add and subtract vectors.
  - Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
  - Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
  - Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
- [College-Ready:] N-VM.5. Multiply a vector by a scalar.
  - Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
  - Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $||c\boldsymbol{v}|| = |c|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

Perform operations on matrices and use matrices in applications.

- [College-Ready:] N-VM.6. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
- [College-Ready:] N-VM.7. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
- [College-Ready:] N-VM.8. Add, subtract, and multiply matrices of appropriate dimensions.

- [College-Ready:] N-VM.9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
- [College-Ready:] N-VM.10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
- [College-Ready:] N-VM.11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
- [College-Ready:] N-VM.12. Work with 2×2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.