Alaska Mathematics Standards
Math Tasks
Grade 3

Three Other Ways

Content Standard	**Mathematical Practices**
3.NBT.1. Use place value understanding to round whole numbers to the nearest 10 or 100. **3.NBT.2.** Use strategies and/or algorithms to fluently add and subtract with numbers up to 1000, demonstrating understanding of place value, properties of operations, and/or the relationship between addition and subtraction.	**1. Make sense of problems and persevere in solving them.** **2. Reason abstractly and quantitatively.** **3. Construct viable arguments and critique the reasoning of others.** **4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.** **5. Use appropriate tools strategically.** **6. Attend to precision.** 7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting. **8. Look for and express regularity in repeated reasoning.**

Task Description

This task provides an opportunity for students to decompose numbers in multiple ways. This will help them easily manipulate numbers as needed in computation, as well as understand the value of digits in numbers when rounding.

Materials:

- base -ten blocks
- attached game boards

****Please see link below for game boards (pg. 20):**
http://tinyurl.com/MathTasks-Grade3-Unit1

PART I

Van de Walle Activity 10.5 Three Other Ways

Students work in groups or pairs. First, they show 463 on their desks with base-ten materials in the standard representation. Next, they find and at least three other ways of representing this number and record on a piece of paper or their math journal. A variation is to challenge students to find a way to show amount with a specific number of pieces. "Can you show 463 with 31 pieces?" (There is more than one way to do this.) Students can get quite involved with finding all the ways to show a three-digit number.

PART II

Students will work in small groups of 2 or 3. Students will cut out the sheets of provided cards. Students will play a game of WAR using these cards. They will pass out all the cards to the players in their group. Each student will flip over the top card in their stack. The students will each calculate the number represented by their card. The student with the largest value will get to collect all the cards that are flipped up. Play will continue until all cards have been flipped up and compared. The student that has the most cards at the end is the winner. Or students can create their own game board and play the game.

Number Talk:

Strategy: Breaking Each Number into Its Place Value

Once students begin to understand place value, this is one of the first strategies they utilize. Each addend is broken into expanded form and like place-value amounts are combined. When combining quantities, children typically work left to right because it maintains the magnitude of the numbers.

For example:

116 + 118 (100 + 10 + 6) + (100 + 10 + 8)	Each addend is broken into its place value.
100 + 100 = 200	100's are combined.
10 + 10 = 20	10's are combined.
6 + 8 = 14	1's are combined.
200 + 20 + 14 = 234	Totals are added from the previous sums.

Below are two Breaking Each Number into Its Place Value Number Talks for you to try with your class:

28 + 11
14 + 35
22 + 15
18 + 31

15 + 27
23 + 18
17 + 25
16 + 27

For additional number talks using this strategy, please see *Number Talks* by Sherry Parrish.

Background Knowledge/Common Misconceptions:
As students enter third grade, they are expected to know numbers up to 1000 (CCSSO, 2010). Here the issue is not one of connecting a count-by-ones concept to a group of 1000, but rather seeing how a group of 1000 can be understood as a group of 10 hundreds as well as 100 tens and 1000 single ones. As a means of introducing thousands as groups of 10 hundreds and also 100 tens, consider the following estimation activity. (Van de Walle, page 160)

Formative Assessment Questions:
• How can you find all the ways to decompose a number?
• When would you need to decompose a number to help you solve a problem?

Differentiation:
Extension
Van de Walle adapted Activity 10.7 Base-Ten Riddles, p. 161

•Base –Ten Riddles can be presented orally or in written form. In either case, students should use base-ten materials to help solve the riddles.

The examples here illustrate a variety of different levels of difficulty. Have students write new riddles when they complete these.

- I have 23 ones and 4 tens. Who am I?
- I have 4 hundreds, 12 tens, and 6 ones. Who am I?
- I have 30 ones and 3 hundreds. Who am I?
- I am 45. I have 25 ones. How many tens do I have?
- I am 341. I have 22 tens. How many hundreds do I have?
- I have 13 tens, 2 hundreds, and 21 ones. Who am I?
- If you put 3 more hundreds with me, I would be 1150. Who am I?
- I have 23 hundreds, 16 tens, and 2 ones. Who am I?

Intervention
Start with showing mixed arrangements of base-ten materials and having students give the base-ten name (4 hundreds, 3 tens, and 8 ones) and the standard name (438). Vary the arrangement from one example to the next by changing only 1 type of piece. That is, add or remove only ones or only tens or only hundreds. It is important for students with disabilities to see counterexamples, so actively point out that some students wrote 200803 for 283, and ask them whether that is correct. These conversations allow students to explore their misunderstandings and focus on the place-value system more explicitly. (Van de Walle, p. 161)

Vocabulary:
Ones
Tens
Hundreds
Decompose
Three-Digit Number

Resources:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010.

Van de Walle, John A., and Lou Ann H. Lovin. Teaching Student-Centered Mathematics: Grades K-3, Volume 1. Pearson, 2006

Alaska Mathematics Standards
Math Tasks
Grade 3

Shake, Rattle, and Roll

Content Standard	**Mathematical Practices**
3.NBT.1. Use place value understanding to round whole numbers to the nearest 10 or 100. **3.NBT.2.** Use strategies and/or algorithms to fluently add and subtract with numbers up to 1000, demonstrating understanding of place value, properties of operations, and/or the relationship between addition and subtraction.	**1. Make sense of problems and persevere in solving them.** **2. Reason abstractly and quantitatively.** **3. Construct viable arguments and critique the reasoning of others.** 4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets. **5. Use appropriate tools strategically.** **6. Attend to precision.** **7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.** **8. Look for and express regularity in repeated reasoning.**

Task Description
This task focuses on rounding as well as the understanding of the value of a digit.

Materials:

- Two six-sided dice
- Calculator
- "Shake, Rattle, and Roll" Recording Sheet

****Please see link below for recording sheet and game directions (pg. 33):**
http://tinyurl.com/MathTasks-Grade3-Unit1

In this task, students play a game with dice that enables them to build estimation and mental math concepts as they practice addition skills and strategies and determine to which multiple of one hundred a given number is nearest.

Task Directions
Students will follow the directions below from the "Shake, Rattle and Roll" Recording Sheet.

This is a two player game that will help you practice your estimation and addition skills. The goal of the game is to be the person with the most points at the end of ten turns.

1. Play with a partner. You will need 3 dice, a recording sheet for each player, and a calculator.
2. Player one rolls the three dice and forms two numbers, the largest possible number and the smallest possible number, as shown below. For example, using the digits 5, 4, and 4 make the numbers 544 and 445. Find the nearest multiple of 100 for each number, and then using mental math, add to find an estimate. Estimated sum = 500 + 400 = 900.
3. Player one records the estimate on the game recording sheet to end round 1. Your partner must agree with your estimation, using a calculator to check if needed.
4. Player two takes a turn, following steps 2 and 3 above.
5. Players take turns for a total of six rounds.
6. After six rounds, each player finds the total sum of their estimates. The player with the higher sum wins the game.

Number Talk:

There are many strategies that can be developed using Number Talks. Finding "Landmark" or "Friendly Numbers" are easy to use when computing mentally (Parrish, 2010). This would be a great strategy to focus on prior to completing the next couple of tasks as students are developing their mental math and rounding.

Strategy: Making Landmark or Friendly Numbers

Landmark or friendly numbers are numbers that are easy to use in mental computation. Multiples of ten, one hundred, one thousand, and so on, as well as twenty-five and fifty, are examples of numbers that fall into this category. Students may adjust one or all addends by adding or subtracting amounts to make a friendly number.

For example:

116 + 118 + 2 116 + 120 = 236 - 2 234	In this example only the 118 is adjusted to make an easy landmark number. The extra 2 that was added on must be subtracted.

Here is a Making Landmark or Friendly Numbers number talk for you to try with your classroom:

19 + 2	39 + 16
19 + 5	28 + 39
19 + 8	59 + 13
19 +12	23 + 49

For additional number talks using this strategy, please see *Number Talks* by Sherry Parrish.

Background Knowledge/Common Misconceptions:

The use of terms like "round up" and "round down" confuses many students. for example, the number 37 would round to 40 or they say it "rounds up". The digit in the tens place is changed from 3 to 4 (rounds up). This misconception is what causes the problem when applied to rounding down. The number 32 should be rounded (down) to 30, but using the logic mentioned for rounding up, some students may look at the digit in the tens place and take it to the previous number, resulting in the incorrect value of 20. To remedy this misconception, students need to use a number line to visualize the placement of the number and/or ask questions such as: "What tens are 32 between and which one is it closer to?" Developing the understanding of what the answer choices are before rounding can alleviate much of the misconception and confusion related to rounding. (Adapted from Ohio Department of Education Model Curricula)

Formative Assessment Questions:

- Explain how you found the closest multiple of one hundred.
- Do you think your estimated sum is higher or lower than the actual sum? Why? How could you check?
- What kinds of situations in life might be easier if you knew how to estimate and add numbers like this?

Differentiation:

Extension

- Ask students to play the game again, estimating to the tens place. Does that change the game? If so, how?
- Play the game with four dice. Students get to choose three of the numbers rolled (or two as an intervention) and players have to get closest to 3000. Whoever is closest (over or under 3000) wins the game. This changes the strategy and allows opportunities for teachers to ask students what they hope to roll on the last roll based on what they have so far.

Intervention

- Use number lines, number charts, and models to help students who are having difficulty determining to which multiple of hundred their number is nearest. Use counting up/counting back to the nearest multiple of hundred and compare the results to determine which multiple of hundred a number is closest.
- Students can play the game using fewer dice, adjusting the game accordingly. Once students become comfortable with fewer dice, they can challenge themselves by playing the game with the required three dice.

Vocabulary:

Round/Rounding
Ones
Tens
Hundreds
Multiple
Sum
Estimate

Resources:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010.

Alaska Mathematics Standards
Math Tasks
Grade 3

The Great Round Up!

Content Standard	**Mathematical Practices**
3.NBT.1. Use place value understanding to round whole numbers to the nearest 10 or 100	**1. Make sense of problems and persevere in solving them.** **2. Reason abstractly and quantitatively.** **3. Construct viable arguments and critique the reasoning of others.** 4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets. **5. Use appropriate tools strategically.** **6. Attend to precision.** 7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting. **8. Look for and express regularity in repeated reasoning.**

Task Description
This task is used to help develop the understanding of the value of a digit as well as to support rounding concepts.

Materials:

- Three Number Cubes
- "The Great Round Up" Recording Sheet

****Please see link below for recording sheet (pg. 38):**
http://tinyurl.com/MathTasks-Grade3-Unit1

Game Instructions
1. Player 1 will toss all three number cubes and make the GREATEST possible 3-digit number with those digits.
2. Player 1 will write his or her number on his or her recording sheet.
3. The player with the GREATEST number in that round will round his or her number to the nearest hundred and record the rounded number in the total column on their recording sheet.
4. All other players will not record a number in the total column for this round.
5. Play will continue for ten rounds.
6. The winner is the player with the greatest total.
7. At the end of the game, students should share their efficient rounding strategies with one another.

Number Talk:

Strategy: Making Landmark or Friendly Numbers

Landmark or friendly numbers are numbers that are easy to use in mental computation. Multiples of ten, one hundred, one thousand, and so on, as well as twenty-five and fifty, are examples of numbers that fall into this category. Students may adjust one or all addends by adding or subtracting amounts to make a friendly number.

For example:

116 + 118 + 2 116 + 120 = 236 - 2 234	In this example only the 118 is adjusted to make an easy landmark number. The extra 2 that was added on must be subtracted.

Here is a Making Landmark or Friendly Numbers number talk for you to try with your classroom:

98 + 5 98 + 13 98 + 34 98 + 52

8 + 4 + 18 18 + 4 +18 28 + 5 + 27 24 + 3 + 48

For additional number talks using this strategy, please see *Number Talks* by Sherry Parrish.

Common Misconceptions:

The use of terms like "round up" and "round down" confuses many students. for example, the number 37 would round to 40 or they say it "rounds up". The digit in the tens place is changed from 3 to 4 (rounds up). This misconception is what causes the problem when applied to rounding down. The number 32 should be rounded (down) to 30, but using the logic mentioned for rounding up, some students may look at the digit in the tens place and take it to the previous number, resulting in the incorrect value of 20. To remedy this misconception, students need to use a number line to visualize the placement of the number and/or ask questions such as: "What tens are 32 between and which one is it closer to?" Developing the understanding of what the answer choices are before rounding can alleviate much of the misconception and confusion related to rounding. (Adapted from Ohio Department of Education Model Curricula)

Formative Assessment Questions:

- How did you determine the greatest number?
- How did you determine which hundred your number was closest to?
- What strategy did you use to determine who won?

Differentiation:

Extension

- Have students create anchor charts for efficient rounding strategies.
- Have students practice rounding to the nearest ten with three dice.
- Have students try rounding to the nearest thousand with four dice.

Intervention

- Students can use number lines or hundreds charts to help them.
- Students can play with two cubes instead of three, rounding to the nearest ten.
- Students can complete the task with a teacher or peer assistant.

Vocabulary:

Round/Rounding
Ones
Tens
Hundreds
Greatest
Three-Digit Number

Resources:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010.

Alaska Mathematics Standards
Math Tasks
Grade 3

What's My Product?

Content Standard

3.OA.1. Interpret products of whole numbers (e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each). For example, show objects in rectangular arrays or describe a context in which a total number of objects can be expressed as 5 × 7.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description
This task allows students to interpret products of whole numbers by creating equal groups with manipulatives.

Materials:
- Colored tiles or two-sided counters
- Something to help organize groups such as paper plates, cups, bowls, etc.
- "What's My Product" recording sheet

****Please see link below for recording sheet (pg. 42):**
http://tinyurl.com/MathTasks-Grade3-Unit2

Part I
Discuss with students how to group objects. Show a container of 20 counters. Discuss with students an easy way to count the total number of counters in the container. Have students arrange the counters into equal groups. As students discuss how to put the 20 counters into groups write their thinking on the board. Explain to students that in a multiplication problem one number represents the number of groups and the other number represents the number of objects in a group.

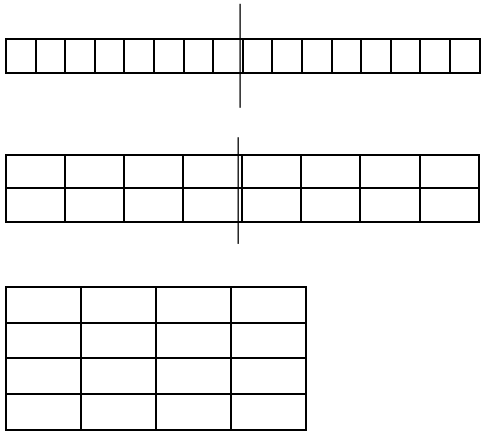
Part II
Provide students with a given set (over 50) of counters or tiles to separate into equal groups. The students will continue to rearrange tiles into different groupings that are equal. As each group is arranged, write a multiplication fact to match the arrangement. Students will record their thinking in the "What's My Product?" recording Sheet.

Number Talk:

Strategy: Doubling and Halving

When students are provided opportunities to build arrays that have the same area and study the patterns of the dimensions, they often will notice a relationship that occurs between the factors or dimensions of the arrays. Consider the number 16. If we were to build all the possible arrays that would make 16 squares, we would have the following dimensions or factor pairs: 1x16, 2x8, 4x4, 8x2, 16x1

In every instance, we still have an area or product of 16, but our dimensions or factors have changed.

	When the 1x16 is halved, the number of rows doubles and the number of columns halve, resulting in 2 x 8. When the 2x8 is halved, the number of rows doubles and the number of columns halve, resulting in a 4x4. Doubling and halving can be continued until a 16x1 array is reached.

This strategy builds on the ease with which students double and halve numbers. We can apply this strategy to several problems.

8 x 25 /2 ( ) X2 4 x 50 /2 ( ) X2 2 x 100 = 200 16 x 16 X2 ( ) /2 32 x 8 X2 ( ) /2 64 x 4 X2 ( ) /2 128 x 2 = 256	The intent of the strategy is to change the problem into a friendly problem to solve. Once the student reaches a point where the solution is easily obtained, then he or she would not continue doubling and halving. Doubling and halving is especially beneficial when multiplying with double digit problems. This can quickly turn the problem into a multiplication problem with a single-digit multiplier.	**Note: some problems do not lend themselves to doubling and halving. This would be an important area for students to investigate.**

Below are two Doubling and Halving Number Talks for you to try with your students.

1 X 16 2 X 8 4 X 4 8 X 2 16 X 1

1 X 12 2 X 6 4 X 3

For more additional number talks using this strategy, please see *Number Talks* by Sherry Parrish.

Common Misconceptions:

Traditionally multiplication tables are emphasized when students begin learning about multiplication. Students are sent home with flash cards without a true understanding of what multiplication is. This way of learning multiplication can be difficult for students to understand. Naturally, students make groups and groups of groups. The creation of groups is a way to find the total of something in the most efficient way. The following activity allows students to build on their natural ability to form groups and learn multiplication without memorizing facts in isolation, but as number facts that can be related to each other in a multitude of ways (Frans van Galen and Catherine Twomey Fosnot, 2007, Context for Learning Mathematics).

Formative Assessment Questions:

- How many ways were you able to organize the number of counters you were given?
- Can you think of another way to organize your counters?
- How can you explain your picture and number sentence in words?

Differentiation:

Extension

- Give students a prime number of counters (29, 31, 47, etc.) and charge students with counting the number of counters through grouping. They cannot count/group by ones! Even though prime numbers are not a third grade standard, the purpose of this extension is for students to create equal groups with an amount "left over". Students can then count by the groups created and add the left over amount to reach the prime number.

Intervention

- Provide smaller numbers of counters and allow students to work with a partner.

Vocabulary:

Product
Whole Number
Multiplication

Resources:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010.

Alaska Mathematics Standards
Math Tasks
Grade 3

Base Ten Multiplication

Content Standard	**Mathematical Practices**
3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*	**1. Make sense of problems and persevere in solving them.** **2. Reason abstractly and quantitatively.** 3. Construct viable arguments and critique the reasoning of others. **4. Model with mathematics.** 5. Use appropriate tools strategically. **6. Attend to precision.** **7. Look for and make use of structure.**

Task Description
In this task students determine the factors of 100 by creating addition and/or multiplication models.

Materials:
- Base Ten Blocks, up to 51 cubes and 60 longs per pair (base 10 template has been provided as well)
- Spinner, numbered 1-9
- "What's My Product" recording sheet
- Packs of 3" X 5" index cards, from 1 to 9 cards per pack, 1 pack per pair
- Overhead Base Ten Blocks (optional)
- Math journal

****Please see link below for base ten block template and spinner template (pg. 81):**
http://tinyurl.com/MathTasks-Grade3-Unit2

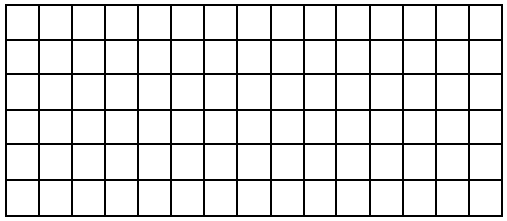
Part I
Ask two volunteers to hold out their hands, palms up.
Place 2 units into each hand. Ask students how they can find the number of units in the four hands. Lead students to counting the units by twos. Record this process as the addition sentence 2 + 2 + 2 + 2 = 8. Elicit that 2 units in each of 4 hands means that there is a total of 8 units. Point out that because the same number, 2, is added 4 times, another way of recording this is with multiplication. Write the multiplication sentence 4 X 2 = 8 on the board. Read it aloud as "Four groups of two equal eight." Have students to suggest ways to record 2 cubes in each of 4 hands.

Part II
Before the task, decide which facts students need the most work with. This will be the number of cards the students will need, this can vary from group to group. Distribute the prearranged packs of index cards to the students. Instruct the students to determine how many cards are in their packs. They will spread out the cards, (the cards are the groups), then spin a spinner. The number that was spun will determine the number of unit blocks on each card. (the number of units in each group).

• *How many cards?*
• *How many* units *on each?*
The students will determine the product and record the number sentence in their math journal.

Part III
Next students will clear off their cards and put an equal number of longs in their place.
• *How many cards?*
• *How many* longs *on each?*
Students will determine the product of the longs and record their number sentence in the math journal. Students will be asked to compare the values they found for the units and for the same number of longs. What did they notice? Repeat the activity several times. (If you spin the same number as before, spin again!)

Number Talk:
Strategy: Repeated Addition or Skip Counting
These are often beginning strategies for students who are just learning multiplication. Help students build connections among these entry-level strategies and multiplication by making the links to multiplication explicit. Connecting the student's strategy to an array model provides an essential visual model for multiplication.

<table>
<tr><th colspan="2">6 x 15</th><th colspan="2">6 x 15 Linked to Multiplication Expressions Using Known Facts</th></tr>
<tr><td>15 + 15 + 15 + 15 + 15 + 15
15 + 15 = 30
30 + 15 = 45
45 + 15 = 60
60 + 15 = 75
75 + 15 = 90</td><td>Students will often think about the problem 6 x 15 as 6 groups of 15 and solve it by adding one number at a time. Record their thinking using the repeated addition, but extend this initial thinking by linking the addition to a multiplication representation with the following strategies.</td><td>(15 + 15) + (15 + 15) + (15 + 15)
2 x 15 = 30
2 x 15 = 30 } 90
2 x 15 = 30
Or
15 + 15 + 15 + 15 + 15 + 15
6 x 10 = 60
6 x 5 = 30 } 90</td><td>Students are often comfortable using doubles and counting by fives and tens in addition. Help them connect this understanding by circling corresponding groups of numbers and recording a correlating multiplication sentence.</td></tr>
<tr><th colspan="2">6 x 15 Linked to an Array Model</th><th colspan="2"></th></tr>
<tr><td>  </td><td>Every multiplication problem can be represented as a rectangle showing rows and columns. For example, 6 x 15 can be represented by an array with 6 rows and 15 columns.</td><td colspan="2"></td></tr>
</table>

Below is a Repeated Addition or Skip Counting Number Talk for you to try with your students.

4 x 9
3 x 5
7 x 6
8 x 9

For more Number Talks using this strategy, please see *Number Talks* by Sherry Parrish.

Common Misconceptions:

When students begin multiplication they are just getting used to counting. Before multiplication, 6 equaled a group of six objects. They also know that 4 equals a group of four objects. However, to think of 4 X 6 they have to think of the group of six as one unit because they need to make four sixes. The four is now used to count groups not objects. This is a hard concept to grasp for students just learning about numbers. Students have to reorganize their thinking (Catherine Twomey Fosnot and Frans van Galen *Contexts for Learning Mathematics*, 2007). This task will give students practice in reorganizing numbers and developing strategies that allow them to make sense of the mathematics.

Formative Assessment Questions:

- What patterns are you noticing?
- What is the relationship between the units and the longs?
- How did you determine your product?
- Could you have determined your product another way?

Differentiation:

Extension

- Instead of using manipulatives to create groups, use digit cards.

Intervention

- Use smaller numbers on the spinner.

Vocabulary:

Pattern
Product
Unit
Long
Multiplication
Fact

Resources:

Fosnot, Catherine Twomey, and Frans van Galen. Contexts for Learning Mathematics. Portsmouth: Heinemann, 2007

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 3

Making Up Multiplication!

Content Standard

3.OA.5. Make, test, support, draw conclusions and justify conjectures about properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.)
• Commutative property of multiplication: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known.
• Associative property of multiplication: 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30.
• Distributive property: Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56.
• Inverse property (relationship) of multiplication and division.

3.OA.6. Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description
In this task, students will learn three different ways a multiplication problem can be written.

Materials:
• Interlocking cubes
• Number strip
• Graph paper
• Two ten-sided dice
• Index cards
• Making up Multiplication, Student recording sheet
• Wipe off boards (optional)

****Please see link below for recording sheets (pg. 154):**
http://tinyurl.com/MathTasks-Grade3-Unit2

Part I-Small Group
Provide each group with an equivalent group/set problem. For example: There are 3 cars. Each has 2 people in it. How many people are there altogether? OR There are 6 fish bowls. Each contains 4 goldfish. How many goldfish are there altogether? OR There are 7 tables. Each table has 4 legs. How many legs are there altogether?

The students can model these and similar types of problems with:
- Interlocking cubes
- Jumps on a number strip
- Drawing a picture
- Creating an array

After the groups have solved their problems and explained their solutions, present the students with the task of making up their own story problems using the number sentence structure above with the different factors and products. The students can write their problems on index cards and swap with other groups. The students can solve the student created problem on a wipe off board or paper showing their strategies and multiplication sentences. Ask the students to label the two factors and one product in the problem. Ask for volunteers to share the problem and how they got the answers.

Did all of the student-created problems work? If some did not work due to wording or values too high, collectively decide how to adjust the problem so it is able to be solved. Return the cards to the owners. Have the owners solve their problem on the back of the index card. The students should also label the factors and product. Collect and review for accuracy and content.

Part II: Exploring different types of story problems(whole group/partners)
First, begin with comparison problems *("times as many"),* where possible, using a context familiar to students. Brainstorm with the class and decide on three separate topics to write their comparison problems. (Possible topic: lunch, pets, toys, types of cards, pencils)

For example: Anna has 3 carrots in her lunchbox and Jessie has *3 times as many* carrots as Anna. How many carrots does Jessie have in her lunchbox?

Have students identify the factors and product in each multiplication problem you solve with the students.

Pose a few questions to the whole group and have students solve collaboratively with their partner and then share their thinking. Ask them to pose their own problems using "*times as many*" in their problem. They can use index cards, wipe off boards or paper to present their own problems to partners to solve. Ask the students to label the two factors and one product in the problem. Share problems, solutions and strategies used.

Next, work with several array problems (equivalent groups). Pose problems such as: The students are lined up in 3 teams. Each team has 6 members. How many students are there altogether? Identify and label the two factors and product.

Students can use graph paper, peg boards, bead strings, or interlocking cubes to create arrays to solve the problems. Students can then create their own array problem for a partner to solve. Ask the students to label the two factors and one product in the problem. Share problems, solutions and strategies used.

Next, work with unknown-factor problems. Allow students to solve a few problems working with their partners, and then ask them to pose their own problems. If you need 1 car for 5 people, how many cars will you need for 15 people? OR If you need 1 fish bowl for 2 goldfish, how many bowls will you need for 18 goldfish? OR If each table seats 4 people, how many tables will you need for 28 people?

Now ask the students to make up word problems using the unknown-factor problem structure. They can use index cards, wipe off boards or paper to present their own problems to partners to solve. Ask the students to label the two factors and one product in the problem. They can use repeated subtraction as well as unknown-factor multiplication.

Finally, ask students to share their problems and solutions. Ask if they could write their answers using a division sign.

Part III

Use the Making up Multiplication, Student recording sheet as evaluation.

Teachers may decide to use this each day to record the student's progress by having the student create and solve the type of multiplication story problem after each session. Then use another copy to assess at the end of the task as record of performance.

- Explain one strategy you can use to solve a multiplication word problem?
- How can you find the unknown-factor in a multiplication problem?
- What is a factor?
- What is a product?

Number Talk:

Strategy: Partial Products

This strategy is based on the distributive property and is the precursor for our standard U.S. algorithm for multiplication – it just keeps the place value intact. The strategy more closely resembles the algorithm when written vertically. When students understand that the factors in a multiplication problem can be decomposed or broken apart into addends, this allows them to use smaller problems to solve more difficult ones. As students invent Partial Product strategies, they can break one or both factors apart.

12 X 15		**12 X 15**	
Horizontal 12x15 12x (10+5) 12x10=120 (12x10) 12x5=60 (12x5) 120+60=180	Vertical 15 x 12 120 + 60 180	Whether the problem is written horizontally or vertically, the fidelity of place value is kept. In this example, the 15 is thought about as (10 + 5) while the 12 is kept whole.	12x15 (4 + 4 + 4) x 15 4 x 15 = 60 4 x 15 = 60 4 x 15 = 60 60 + 60 + 60 =180 12 x 15 (10 + 2) x (10 + 5) 10 x 10 = 100 10 x 5 = 50 2 x 10 = 20 2 x 5 = 10 100 + 50 + 20 + 10 = 180	This time the 12 is broken apart into (4 + 4 + 4) and the 25 is kept whole. The 12 could have been broken into (10 +2) or any other combination that would have made the problem accessible. Both factors can be broken apart, and as numbers become larger, students often use this method until they become more confident in multiplying with larger quantities. It is difficult for some students to keep up with all of the parts of the problem, especially when trying to use this strategy without paper and pencil.

Below are two Partial Products Number Talks for you to try with your students.

2 x 7 4 x 7 4 x 8 3 x 8 8 x 7	2 x 15 3 x 15 6 x 5 6 x 10

For more Number Talks using this strategy, please see *Number Talks* by Sherry Parrish.

Common Misconceptions:
In part II of this task, students will begin exploring different types of multiplication word problems and begin looking at comparison problems. Comparison problems use the terms "times as many". It is important for the teacher to recognize this as a comparison problem, but should not be taught as a key phrase for students to learn. Students should build their understanding of the problem by analyzing what is happening in the problem.

Formative Assessment Questions:

- Explain one strategy you can use to solve a multiplication word problem?
- How can you find the unknown-factor in a multiplication problem?
- What is a factor?
- What is a product?

Differentiation:

Extension
- Give the student the product and ask them to find out how many different factors could be used to solve the problem. Which factors would be the most likely? Explain the reasoning and strategy used.

Intervention

- Guided practice that simulates the task, done ahead of time, will enable students to develop problem solving strategies.
- Using manipulatives instead of graph paper.
- Use Making Up Multiplication, Student Response sheet with problem already created. The student will solve the problem and label the parts of the number sentence only.

Vocabulary:
Word Problem
Factor
Product
Multiplication
Comparison Problem
Array Problem
Unknown-Factor Problem

References:
Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 3

Count Me In!

Content Standard

3.MD.7. Recognize area as an attribute of plane figures and understand concepts of area measurement.
a. A square with side length 1 unit is said to have "one square unit" and can be used to measure area.
b. Demonstrate that a plane figure which can be covered without gaps or overlaps by n (e.g., 6) unit squares is said to have an area of n (e.g., 6) square units.

3.MD.8. Measure areas by tiling with unit squares (square centimeters, square meters, square inches, square feet, and improvised units).

3.MD.9. Relate area to the operations of multiplication and addition.
a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. *For example, after tiling rectangles, develop a rule for finding the area of any rectangle.*

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description
In this task, students create area models and label them with appropriate dimensions.

Materials:

- grid paper
- Post-it notes (square), color tiles, or construction paper squares
- math journal/learning log

****Please see link below for "count me in" task sheet (pg. 44):**
http://tinyurl.com/MathTasks-Grade3-Unit3

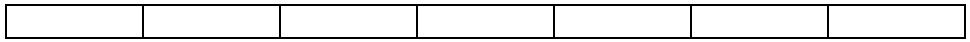
Part I
Students use color tiles, post-its, or colored squares of paper to create a rectangle representing an area of 24. Have students share all the different dimensions that could be used to create rectangles. (1x24, 2x12, 3x8, or 4x6)

As a class, lead a discussion using the questions below to assist students in understanding the relationship between addition and multiplication when using arrays.

● While your figure may look different from someone else's figure, how do both figures show the same area? (This relates back to the Cover Me scaffolding task.)
● Write the following dimensions on the board representing the variety of arrays made by students: 1 by 24; 2 by 12; 3 by 8; and 4 by 6. (Be certain not to use the multiplication symbol, x)
● What math could be used to quickly determine the area of each figure? This question should lead the students to saying that they could use skip counting or some may respond that they would multiply the number of columns and number of rows.
● Have students use both repeated addition and multiplication to show how the area of the figure could be found. Record students responses for all to see.
● Provide student pairs with grid paper and a product. Use products that yield more than one set of dimensions/factors.
● Student pairs draw all the ways their product can be shown using rectangles and square units such as color tiles, square post-its, or colored paper squares.
● Pairs record equations for each figure using repeated addition and multiplication.

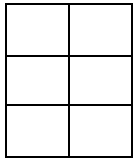
Example:

Product = 6



1 + 1 + 1 + 1 + 1 + 1 = 6

6 x 1 = 6

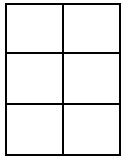


2 + 2 + 2 = 6
3 x 2 = 6

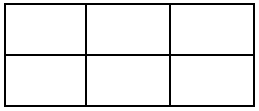
Big Ideas

● Area is related to addition in that the square units can be counted two ways: based on columns or length along with width or rows. This can be done using skip counting which is a form of addition.
● The most efficient way to determine a figure's area is to multiply the number of squares in one column by the number of squares in one row..
● Consider the product 12. The commutative property of multiplication can be seen by turning the figures to represent 3 groups of 4 or 4 groups of 3. Both yield the same area but the figure has different lengths and width depending on the figure's orientation. This will make the connection to the commutative property of multiplication.

Example:



2 + 2 + 2 = 6
2 x 3 = 6

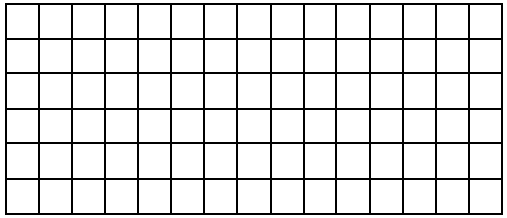


3 + 3 = 6
3 x 2 = 6

Part II
Students complete the "Count Me In" task using recording form. Have students share the different rectangles created and how they are similar/different.

Number Talk:
Strategy: Repeated Addition or Skip Counting
These are often beginning strategies for students who are just learning multiplication. Help students build connections among these entry-level strategies and multiplication by making the links to multiplication explicit. Connecting the student's strategy to an array model provides an essential visual model for multiplication.

<table>
<tr><th colspan="2">6 x 15</th><th colspan="2">6 x 15 Linked to Multiplication Expressions Using Known Facts</th></tr>
<tr>
<td>15 + 15 + 15 + 15 + 15 + 15

15 + 15 = 30
30 + 15 = 45
45 + 15 = 60
60 + 15 = 75
75 + 15 = 90</td>
<td>Students will often think about the problem 6 x 15 as 6 groups of 15 and solve it by adding one number at a time. Record their thinking using the repeated addition, but extend this initial thinking by linking the addition to a multiplication representation with the following strategies.</td>
<td>(15 + 15) + (15 + 15) + (15 + 15)
2 x 15 = 30
2 x 15 = 30 } 90
2 x 15 = 30
Or
15 + 15 + 15 + 15 + 15 + 15
6 x 10 = 60 } 90
6 x 5 = 30</td>
<td>Students are often comfortable using doubles and counting by fives and tens in addition. Help them connect this understanding by circling corresponding groups of numbers and recording a correlating multiplication sentence.</td>
</tr>
<tr><th colspan="2">6 x 15 Linked to an Array Model</th></tr>
<tr>
<td>  </td>
<td>Every multiplication problem can be represented as a rectangle showing rows and columns. For example, 6 x 15 can be represented by an array with 6 rows and 15 columns.</td>
</tr>
</table>

Below is a Repeated Addition or Skip Counting Number Talk for you to try with your students.

7 x 3 5 x 9 4 x 6 8 x 4

For more Number Talks using this strategy please see *Number Talks* by Sherry Parrish.

Common Misconceptions:
Students may think that different shapes made with the same units have different areas as well. This is due to lack of experience in developing conservation on area. (*Math Misconceptions: From Misunderstanding to Deep Understanding*, Bamberger, Oberdorf, and Shultz-Ferrel 2010). Using activities such as the one presented in the lesson with tangrams and arrays will provide the needed spatial experience for students to develop this understanding.

Formative Assessment Questions:
- What is the connection between skip-counting/addition and multiplication?
- Which one is better to use and why?
- Can the same areas look different? Why or why not?
- What is the commutative property of multiplication and how does it relate to area?
- What is the relationship between a product and a sum?
- Can an area measurement have the same area but different factors? How does that relate back to the tangram task?
- What would happen if we took some of the post-its or blocks away? Would we still have an accurate area measurement of the plane figure? Explain.

Differentiation:
Extension
- Provide students with 1cm grid paper. Given a two different number products, (one prime number product such as 7 and one composite product such as 15) have students create all the arrays possible for each product. Have students use drawings to illustrate the difference between prime and composite number.

Intervention
- http://www.brainpopjr.com/math/measurement/area/grownups.weml
  Have students view the short video clip and discuss as needed.

Vocabulary:
Rectangle
Array
Product
Factor
Column
Row
Sum
Commutative Property

References:

Bamberger, Honi J., Christine Oberdorg, Karren Schultz-Ferrell. Math Misconceptions, PreK-Grade 5: From Misunderstanding to Deep Understanding. Portsmouth: Heinemann, 2010
Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 3

Olympic Cola Display

Content Standard

3.MD.9. Relate area to the operations of multiplication and addition.
c. Use area models (rectangular arrays) to represent the distributive property in mathematical reasoning. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description

In this task, students will use their understanding of area models to represent the distributive property to solve problems associated with an Olympic cola display.

Materials:

- Act 1 picture -Olympic Cola Display
- Pictorial representations of the display
- Student recording sheet

****Please see link below for Act 1 picture, pictorial representation and student recording sheet (pg. 72):**
http://tinyurl.com/MathTasks-Grade3-Unit3

Act 1 –Whole Group -Pose the conflict and introduce students to the scenario by showing Act I picture.. (Dan Meyer http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/

"Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible."

- Show Act 1 picture to students.
- Ask students what they noticed in the picture, what they wonder about, and what questions they have about what they saw in the picture. Do a think-pair-share so that students have an opportunity to talk with each other before sharing questions with the whole group.
- Share and record students' questions. The teacher may need to guide students so that the questions generated are math-related.

Anticipated questions students may ask and wish to answer: (*Main question(s) to be investigated)

- How many 12 packs of Coke are there?
- *How many 12 packs are there in the display?

- *How many cans of soda is that?
- How tall is it?
- How wide is it?
- What is the area of the front of the display?
- How much time did it take to make that display?
- Where is it?
- How many cans of each kind of soda are in the display?
- What are the dimensions of the display?

●Once students have their question, ask the students to estimate answers to their questions (think-pair-share). Students will write their best estimate, then write two more estimates –one that is too low and one that is too high so that they establish a range in which the solution should occur. Students should plot their three estimates on an empty number line. Note: As the facilitator, you may choose to allow the students to answer their own posed questions, one question that a fellow student posed, or a related question listed above. For students to be completely engaged in the inquiry-based problem solving process, it is important for them to experience ownership of the questions posed.

Important note:
Although students will only investigate the main question(s) for this task, it is important for the teacher to not ignore student generated questions. Additional questions may be answered after they've found a solution to the main question, or as homework or extra projects.

Act 2 –Student Exploration -Provide additional information as students work toward solutions to their questions. Dan Meyer: http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/
"The protagonist/student overcomes obstacles, looks for resources, and develops new tools."

● During Act 2, students decide on the facts, tools, and other information needed to answer the question(s) (from Act1). When students decide what they need to solve the problem, they should ask for those things. It is pivotal to the problem solving process that students decide what is needed without being given the information up front.
● Students need the opportunity to work with manipulatives on their own or with a partner in order to develop the understanding of multiplication. From the manipulatives, students will be able to move to pictorial representations of the display (attached), then more abstract representations (such as sketches), and finally to abstract representation of multiplication using numbers. It is important to remember that this progression begins with concrete representations using manipulatives.
● The teacher provides guidance as needed during this phase. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin. Questioning is an effective strategy that can be used, with questions such as:
- What is the problem you are trying to solve?
- What do you think affects the situation?
- Can you explain what you've done so far?
- What strategies are you using?
- What assumptions are you making?
- What tools or models may help you?
- Why is that true?
- Does that make sense?

Additional Information for Act 2
● It is during Act 2 that you may provide the students with the pictorial representation of the display that is attached.

Act 3 –Whole Group –Share solutions and strategies.

- Students to present their solutions and strategies and compare them.
- Lead discussion to compare these, asking questions such as:
  - How reasonable was your estimate?
  - Which strategy was most efficient?
  - Can you think of another method that might have worked?
  - What might you do differently next time?

Act 4, The Sequel -"The goals of the sequel task are to a) challenge students who finished quickly so b) I can help students who need my help. It can't feel like punishment for good work. It can't seem like drudgery. It has to entice and activate the imagination." Dan Meyer: http://blog.mrmeyer.com/2013/teaching-with-three-act-tasks-act-three-sequel/

For Act 4, share ideas (see extensions) or reference other student-generated questions that could be used for additional classwork, projects or homework.

Number Talk:

Strategy: Partial Products

This strategy is based on the distributive property and is the precursor for our standard U.S. algorithm for multiplication – it just keeps the place value intact. The strategy more closely resembles the algorithm when written vertically. When students understand that the factors in a multiplication problem can be decomposed or broken apart into addends, this allows them to use smaller problems to solve more difficult ones. As students invent Partial Product strategies, they can break one or both factors apart.

12 X 15		**12 X 15**	
Horizontal 12x15 12x (10+5) 12x10=120 (12x10) 12x5=60 (12x5) 120+60=180	Vertical 15 x 12 120 + 60 180	Whether the problem is written horizontally or vertically, the fidelity of place value is kept. In this example, the 15 is thought about as (10 + 5) while the 12 is kept whole.	12x15 (4 + 4 + 4) x 15 4 x 15 = 60 4 x 15 = 60 4 x 15 = 60 60 + 60 + 60 =180 12 x 15 (10 + 2) x (10 + 5) 10 x 10 = 100 10 x 5 = 50 2 x 10 = 20 2 x 5 = 10 100 + 50 + 20 + 10 = 180	This time the 12 is broken apart into (4 + 4 + 4) and the 25 is kept whole. The 12 could have been broken into (10 +2) or any other combination that would have made the problem accessible. Both factors can be broken apart, and as numbers become larger, students often use this method until they become more confident in multiplying with larger quantities. It is difficult for some students to keep up with all of the parts of the problem, especially when trying to use this strategy without paper and pencil.

Below are two Partial Product Number Talks for you to try with your students.

8 x 5 8 x 2 8 x 50 8 x 56	2 x 45 4 x 45 2 x 40 2 x 5 8 x 45

For more Number Talks using this strategy please see *Number Talks* by Sherry Parrish.

Common Misconceptions:

A common misconception is that students should learn their multiplication tables 0-12 in order. Van de Walle states that students need to see multiplication as patterns and use different strategies in determining the product of factors such as the base-ten model and partial products.

Formative Assessment Questions:

- What partial products did you create?
- What organizational strategies did you use?
- What are the dimensions of your array(s)?
- What product/area does your model represent?

Differentiation:

Extension

- Give students a base-ten block array or a drawing of an array and have them determine the product and its factors.
- Have students create their own display, build it with base 10 blocks or connecting cubes, and then trade seats with a neighbor to determine the factors and find the product.
- Have students use an array to write/solve division problems.

Intervention

- Begin with much smaller arrays, such as 2 x 3, 3 x 4, and 2 x 6. Have students describe the dimensions and area of each array. Then connect dimensions and area to the actual multiplication sentence.
- Use grid paper and allow students to place the base-ten blocks onto the grid paper first and then to count the grid squares as part of their calculations.
- If necessary, allow students to use a times table chart or other cueing device if full mastery of the basic multiplication facts has not yet been attained.

Vocabulary:

Tall	Dimensions	Array
Wide	Partial Products	Estimate
Area		

References:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Van de Walle, John A., and Lou Ann H. Lovin. Teaching Student-Centered Mathematics: Grades K-3, Volume 1. Pearson, 2006

Alaska Mathematics Standards
Math Tasks
Grade 3

Read All About It!

Content Standard

3.OA.8. Solve and create two-step word problems using any of the four operations. Represent these problems using equations with a symbol (box, circle, question mark) standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
3.MD.8. Measure areas by tiling with unit squares (square centimeters, square meters, square inches, square feet, and improvised units).
3.MD.9. Relate area to the operations of multiplication and addition.
a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. *For example, after tiling rectangles, develop a rule for finding the area of any rectangle.*
b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
c. Use area models (rectangular arrays) to represent the distributive property in mathematical reasoning. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$.
d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. *For example, the area of a 7 by 8 rectangle can be determined by decomposing it into a 7 by 3 rectangle and a 7 by 5 rectangle.*

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description
This task provides students with experiences solving multistep real world problems.

Materials:

- "Read All About it" task sheet
- 1 inch color tiles
- 8 ½ x 11 inch paper

****Please see link below for task sheet (pg. 104):**
http://tinyurl.com/MathTasks-Grade3-Unit3

The teacher will present the students with the following problem solving task:

The 3rd grade class at Georgia Elementary School wanted to go on a field trip to a soda factory. The trip will cost $100. The students decided to write a class newspaper and sell it to the kids at their school. Each of the 20 students will be given a 16 inch square for his/her article in the newspaper. How many pages long will the newspaper be if they used paper that was 8 ½ x 11 inches? Will there be enough room for additional graphics on the pages once the articles have been written? How did you determine this?

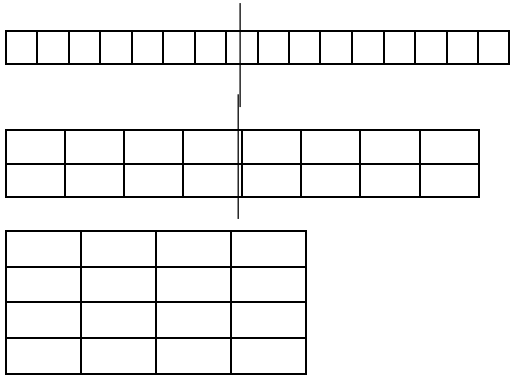
Students should be allowed to use 1 inch color tiles as well as sheets of paper to complete the task. They should show their solution using pictures, numbers and words.

Number Talk:

Strategy: Doubling and Halving

When students are provided opportunities to build arrays that have the same area and study the patterns of the dimensions, they often will notice a relationship that occurs between the factors or dimensions of the arrays. Consider the number 16. If we were to build all the possible arrays that would make 16 squares, we would have the following dimensions or factor pairs: 1x16, 2x8, 4x4, 8x2, 16x1

In every instance, we still have an area or product of 16, but our dimensions or factors have changed.

	When the 1x16 is halved, the number of rows doubles and the number of columns halve, resulting in 2 x 8. When the 2x8 is halved, the number of rows doubles and the number of columns halve, resulting in a 4x4. Doubling and halving can be continued until a 16x1 array is reached.

This strategy builds on the ease with which students double and halve numbers. We can apply this strategy to several problems.

8 x 25 /2 ( ) X2 4 x 50 /2 ( ) X2 2 x 100 = 200	The intent of the strategy is to change the problem into a friendly problem to solve. Once the student reaches a point where the solution is easily obtained, then he or she would not continue doubling and halving.	16 x 16 X2 ( ) /2 32 x 8 X2 ( ) /2 64 x 4 X2 ( ) /2 128 x 2 = 256	Doubling and halving is especially beneficial when multiplying with double digit problems. This can quickly turn the problem into a multiplication problem with a single-digit multiplier.
Note: some problems do not lend themselves to doubling and halving. This would be an important area for students to investigate.			

Below are two Doubling and Halving Number Talks for you to try with your students.

1 x 56
2 x 28
4 x 14
8 x 7

1 x 40
2 x 20
4 x 10
8 x 5

For additional number talks using this strategy please see *Number Talks* by Sherry Parrish.

Common Misconceptions:

The single most important principle for improving the teaching of mathematics is to allow the subject of mathematics to be problematic for students. That is, students solve problems not to apply mathematics but also to learn new mathematics. When students engage in well-chosen problem-based tasks and focus on the solution methods, what results is the new understanding of the mathematics embedded in the task. When students are actively looking for relationships, analyzing patterns, finding out which methods work and which don't, justifying results, or evaluating and challenging the thoughts of others, they are necessarily and optimally engaging in reflective thought about the ideas involved (Van de Walle, *Teaching Student Centered Mathematics*).

Formative Assessment Questions:

- How did you determine the number of pages needed?
- Is there another way you could have solved this?
- Did you find a pattern when you were solving this?
- How does your knowledge of area help you solve this problem?

Differentiation:

Extension

- The students could determine the cost of producing the paper, and how many copies should be sold and at what price, in order to reach their goal.

Intervention

- Decrease the number of students that are writing articles.
- Use this task in a guided small group.

Vocabulary:

Pattern
Inches
Solution

Resources:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Van de Walle, John A., and Lou Ann H. Lovin. Teaching Student-Centered Mathematics: Grades K-3, Volume 1. Pearson, 2006

Alaska Mathematics Standards
Math Tasks
Grade 3

What Makes A Shape?

Content Standard	**Mathematical Practices**
3.G.1. Categorize shapes by different attribute classifications and recognize that shared attributes can define a larger category. Generalize to create examples or non-examples.	**1. Make sense of problems and persevere in solving them.** **2. Reason abstractly and quantitatively.** **3. Construct viable arguments and critique the reasoning of others.** **4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.** **5. Use appropriate tools strategically.** **6. Attend to precision.** **7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.** 8. Look for and express regularity in repeated reasoning.

Task Description
In this task, students begin the process of exploring shapes for their many attributes and use critical vocabulary to describe and compare those shapes through higher-level thinking skills.

Materials:

- Glue and scissors
- *The Greedy Triangle* by Marilyn Burns or other book about shape attributes
- "What Makes a Shape? Shapes for Sorting" student sheet, copied on colored paper
- "What Makes a Shape? Venn Diagram" student recording sheet, copied on white paper

****Please see link below for student sheet and Venn Diagram (pg. 38):**
http://tinyurl.com/MathTasks-Grade3-Unit4

Part I
Teachers may want to begin this task by reading a book about shape attributes such as *The Greedy Triangle*. While reading, questions should be posed to the students that lead to the discovery of shape attributes –their similarities and differences. A list of attributes may be generated on the board throughout the reading or each student may be asked to keep a list of attributes. These words may already be on an anchor chart from the previous task.

Part II
When students are working in pairs, the teacher should monitor the questioning and discussion between the students, and if necessary, model a discussion prior to or during the work time. Students will follow the directions below from the "What Makes a Shape?" student recording sheet:

1. Cut out the shapes below.
2. Sort the shapes in different ways. (Use the list of attributes to help you think of different ways to sort the shapes.)
   - Students may sort shapes by such attributes as number of vertices, or size of angles. Responses should clearly indicate how the shapes were grouped. Exemplary responses would include the use of a graphic organizer, explanations or labels that are clear, and appropriate mathematical vocabulary.
3. Choose two attributes and label the Venn diagram.
4. Sort your shapes in the Venn diagram leaving any shapes that don't fit outside of the Venn diagram.
5. Once you have checked your work, glue the shapes on the Venn diagram.
6. Write to explain your thinking and to describe any observations you made.

Once students have completed their Venn diagrams, encourage them to share their work. A few students can be selected during the work time to share their work and explain their thinking. Or if students have had experience sharing their work, they can be placed in small groups and each student can share their work with their group.

Number Talk:

Even though this task involves a geometry standard, it is still important to practice number talks daily. There is an example of a number talk appropriate for 3rd grade below. However, feel free to choose or create a number talk that is relevant and/or needed for the students in your classroom.

Strategy: Keeping a Constant Difference (Subtraction):

As students begin to understand subtraction as the difference between two quantities, they can investigate what occurs if both numbers are changed by the same amount. Allowing students to explore this relationship with smaller problems such as 5-3 is a way to help them build this understanding. If 5 and 3 are both changed by +2, the problem 7-5 will result. Notice there is still a difference of 2. What if we removed 2 from each number in the problem 5-3? We would then create the problem 3-1, which still results in a difference of 2. Adding or subtracting the same quantity from both the subtrahend and minuend maintains the difference between the numbers. Manipulating the numbers in this way allows the student to create a friendlier problem without compromising the result.

123 - 59 123 + 1 = 124 - 59 + 1 = - 60 64	Both numbers have been adjusted by +1, which makes a problem with an easy 10. Deciding on the amount to subtract or add to adjust the problem is a big decision. For instance, would it have been helpful to adjust each number by -1? This would have created the problem 122 – 58, which is not an easier problem to solve.

Below are two Keeping a Constant Difference Number Talks for you to try with your students.

14 – 10	61 – 29
13 – 9	62 – 30
14 – 7	59 – 27
15 - 6	49 – 17

For more Number Talks using this strategy please see *Number Talks* by Sherry Parrish.

Common Misconceptions:
Students may identify a square as a "non-rectangle" or a "non-rhombus" based on limited images they see. They do not recognize that a square is a rectangle because it has all of the properties of a rectangle. They may list properties of each shape separately, but not see the interrelationships between the shapes. For example, students do not look at the properties of a square that are characteristic of other figures as well. Using straws to make four congruent figures have students change the angles to see the relationships between a rhombus and a square. As students develop definitions for these shapes, relationships between the properties will be understood.

Formative Assessment Questions:
- How could you describe this figure in relationship to another figure?
- Why did you place the figure here? (Indicate a section of the Venn diagram.)
- How do you know this shape is in the correct place?
- Choose one plane figure and tell me how it is used in the world and why its attributes are important in that use.
- Can you choose a shape not included and tell me where it would fit on your paper and why?

Differentiation:

Extension
- Have students select different ways to compare/contrast the shapes, then compare their way of sorting with another student.
- Use solid figures instead of plane figures.
- Incorporate a writing opportunity by having students write a compare/contrast paragraph using 2 shapes.

Intervention
- Select a smaller sample of shapes. Provide the labels and a graphic organizer for students or do the reverse in a discovery model and set out some of the shapes in the organizer and let students determine the correct labels, then sort the remaining shapes.
- If students are having difficulty participating in productive conversations, the teacher should model using think-alouds or self-questioning strategies.

Vocabulary:
Vertices
Angle
Similarities
Differences

References:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 3

Pentomino Perimeters

Content Standard	**Mathematical Practices**
3.MD.9. Relate area to the operations of multiplication and addition. **3.MD.10.** Solve real world and mathematical problems involving perimeters of polygons, including: • finding the perimeter given the side lengths, • finding an unknown side length, • exhibiting rectangles with the same perimeter and different areas, • exhibiting rectangles with the same area and different perimeters.	**1. Make sense of problems and persevere in solving them.** 2. Reason abstractly and quantitatively. **3. Construct viable arguments and critique the reasoning of others.** **4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.** **5. Use appropriate tools strategically.** **6. Attend to precision.** **7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.** 8. Look for and express regularity in repeated reasoning.

Task Description
In this task, students will explore area and perimeter and their relationship through the use of pentominos.

Materials:

- "Pentominos Perimeters" student recording sheet (2 pages)
- Pentominos (may use paper copies)
- *Racing Around*, by Stuart J. Murphy, or similar book about finding perimeter

****Please see link below for recording sheets and pentomino paper copies (pg. 120):**
http://tinyurl.com/MathTasks-Grade3-Unit4

Part I
One way to introduce the concept of perimeter is to read *Racing Around*, by Stuart J. Murphy, or a similar book about finding perimeter.

Part II
Students will follow the directions below from the "Pentomino Perimeters" student recording sheet.
1. In each box below, choose three pentominos and create a polygon. Trace your polygon in the box.
2. Find the area and perimeter of each polygon. Be sure to include the correct label for each measure.
3. Write how you found the area and perimeter of your polygons.
4. Explain what you noticed about the areas and perimeters of your polygons.

CHALLENGE:
• Using 3 pentomino pieces, what is the longest perimeter you can make? Sketch it below and explain how you know it has the longest possible perimeter.
• Using 3 pentomino pieces, what is the largest area you can make? Sketch it below and explain how you know it has the largest possible area.

Number Talk:
Even though this task involves a geometry standard, it is still important to practice number talks daily. There is an example of a number talk appropriate for 3rd grade below. However, feel free to choose or create a number talk that is relevant and/or needed for the students in your classroom.

Strategy: Adding Up
This strategy allows students to build on their strength with addition by adding up from the subtrahend (the number being subtracted) to the minuend (the whole). When students begin to understand that subtraction is finding the difference between two quantities, they realize that they can add up to compute that distance. Even with large number the Adding Up strategy is effective and efficient. Some students will need to jump to every number. Help them think about jumps to get to the nearest ten or friendly number. The larger the jumps, the more efficient the strategy will be. Conversations that encourage students to focus on patterns with combinations that make ten, one hundred, one thousand, and so on, are important for navigating efficient jumps.

123 – 59 + 1 + 40 + 23 = 64 59 60 100 123 123 – 59 59 + 1 = 60 60 + 40 = 100 100 + 23 = 123 1 + 40 + 23 = 64	As students begin to use this strategy, it is helpful to record their thinking on an open number line to show that they are tracking the distance between the subtrahend and minuend. Notice how the student first moves to the nearest 10 and then navigates from this point. Eventually, students will be able to record their thinking without a physical number-line model.

Below are two Adding Up Number Talks for you to try with your students.

20 – 15
20 – 14
20 – 9
20 - 8

90 – 79
90 – 74
90 – 49
90 – 44

For additional Number Talks using this strategy please see *Number Talks* by Sherry Parrish.

Common Misconceptions:
Students may confuse perimeter and area when they measure the sides of a rectangle and then multiply. They think the attribute they find is length, which is perimeter. Pose problems situations that require students to explain whether they are to find the perimeter or area.

Formative Assessment Questions:
- How does the area compare to the perimeter of this shape?
- What units are used to measure each polygon? Why?
- What generalizations can you make about the relationship of perimeter and area of shapes?
- Look at the shapes of other classmates. How does your area and perimeter differ from theirs? If there is a difference, why is it so?

Differentiation:
Extension
- Ask students to complete the challenge on the student recording sheet.
- Challenge students to find 4 pieces that create a 4 x 5 rectangle or 5 pieces that form a 5 x 5 square. For more extension activities, see the following web site: http://people.rit.edu/mecsma/Professional/Puzzles/Pentominoes/P-A.html

Intervention
- Have students copy and draw the square units inside a pentomino piece and then label the perimeter and area for further understanding.
- Use a visual model for students to copy.

Vocabulary:
Area
Perimeter
Polygon
Compare

References:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 3

Guess Who's Coming To Dinner?

Content Standard

3.MD.9. Relate area to the operations of multiplication and addition.
3.MD.10. Solve real world and mathematical problems involving perimeters of polygons, including:
• finding the perimeter given the side lengths,
• finding an unknown side length,
• exhibiting rectangles with the same perimeter and different areas,
• exhibiting rectangles with the same area and different perimeters.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description
In this task, students will manipulate squares to alter the perimeter of given shapes in order to maximize seating potential. Students will then determine the size table cloth needed for the table of their choice.

Materials:

• *Spaghetti and Meatballs For All* by Marilyn Burns or similar book about perimeter
• "Guess Who's Coming to Dinner?" student recording sheet
• 8 colored squares per group (about 2-inch squares)
• 1 large paper per group (about 18 x 24)

****Please see link below for recording sheet and student example (pg. 138):**
http://tinyurl.com/MathTasks-Grade3-Unit4

Part I
As an introduction to this task, read *Spaghetti and Meatballs for All*. In the story, relatives come to dinner and begin rearranging tables which results in losing seating places. After reading the book, have groups use the squares to model some of the events in the book. Discuss changes in area and/or perimeter caused by the moves.

Part II
Students will follow the directions below from the "Guess Who's Coming to Dinner?" student recording sheet.

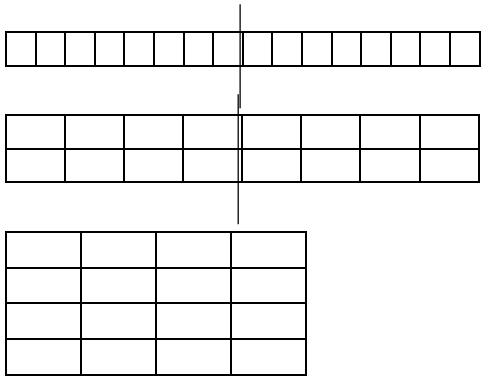
Pretend that four people live at your house (Your mom, dad, sister, and you). Aunt Sue, Uncle John and their six children (Jamal, Kevin, Carl, Annie, Stephanie, and Maxine) are coming for dinner. Uncle Kenny is coming, too. He is bringing his wife (Aunt Jenny) and four kids (Earl, Charles, Jasmine and Justine). Mom has six square folding tables she can use but you don't have to use all of them. (Each folding table seats four, one on each side.)You can put two or more of the folding tables together to form a rectangle if you like.

Job #1:
Your job is to work with a partner to decide on a seating arrangement that is best for your family and guests. When finished, draw a picture of the table arrangement and label each place to show who will be sitting there.
Mom has the following rules:
• There should be no empty seats.
• There must be at least one grown-up at each table.
Write a few sentences to describe what happened to the perimeter as tables were pushed together. Then explain why the arrangement you chose is the best possible arrangement.

Part III
Job #2:
Next, you need to determine what size tablecloth your mom needs in order to cover the table. Each side of each square is 3 feet long. The table cloth should be a perfect fit.

<u>Number Talk:</u>
Strategy: Doubling and Halving
When students are provided opportunities to build arrays that have the same area and study the patterns of the dimensions, they often will notice a relationship that occurs between the factors or dimensions of the arrays. Consider the number 16. If we were to build all the possible arrays that would make 16 squares, we would have the following dimensions or factor pairs:

1x16, 2x8, 4x4, 8x2, 16x1
In every instance, we still have an area or product of 16, but our dimensions or factors have changed.



When the 1x16 is halved, the number of rows doubles and the number of columns halve, resulting in 2 x 8.

When the 2x8 is halved, the number of rows doubles and the number of columns halve, resulting in a 4x4.

Doubling and halving can be continued until a 16x1 array is reached.

This strategy builds on the ease with which students double and halve numbers. We can apply this strategy to several problems.

8 x 25 /2 ( ) X2 4 x 50 /2 ( ) X2 2 x 100 = 200	The intent of the strategy is to change the problem into a friendly problem to solve. Once the student reaches a point where the solution is easily obtained, then he or she would not continue doubling and halving. Doubling and halving is especially beneficial when multiplying with double digit problems. This can quickly turn the problem into a multiplication problem with a single-digit multiplier.
16 x 16 X2 ( ) /2 32 x 8 X2 ( ) /2 64 x 4 X2 ( ) /2 128 x 2 = 256	**Note: some problems do not lend themselves to doubling and halving. This would be an important area for students to investigate.**

Below are two Doubling and Halving Number Talks for you to try with your students.

1 x 32
2 x 16
4 x 8
8 x 4
16 x 2
32 x 1

1 x 48
2 x 24
4 x 12
8 x 6
16 x 3

For additional number talks using this strategy please see *Number Talks* by Sherry Parrish.

Common Misconceptions:
Students may confuse perimeter and area when they measure the sides of a rectangle and then multiply. They think the attribute they find is length, which is perimeter. Pose problems situations that require students to explain whether they are to find the perimeter or area.

Formative Assessment Questions:

- How does the area compare to the perimeter of this shape?
- How does combining or pulling apart shapes affect the perimeter and area of your pieces?
- What happens when you combine squares?
- What strategies are you using to make sure each guest has a seat?

Differentiation:

Extension

- Ask students work with a total of 24 dinner guests and 8 square tables.
- Challenge students to find more than one way to solve the problem.
- Ask students to describe how area and perimeter are alike and/or different.

Intervention

- As students try out a possible solution, have them trace the squares on a separate piece of paper and label the area and length of sides to determine the perimeter. Continue with this until the perimeter matches the number of guests. Then have students use name cards to move the guests around until a suitable solution is found.

Vocabulary:
Area
Perimeter
Seating arrangement
Combine

References:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 3

Comparing Fractions

Content Standard	**Mathematical Practices**
3.NF.3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. b. Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual fraction model).	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting. 8. Look for and express regularity in repeated reasoning.

Task Description
Students will create models of fractions that they can manipulate to find equivalent fractions

Materials:

- Comparing Fractions task sheet
- 9" x 12" sheets of paper in six different colors (cut into 1" x 12" strips) Each child will need 6 strips, one of each color.
- Scissors

****Please see link below for task sheet (pg. 27):**
http://tinyurl.com/MathTasks-Grade3-Unit5

Part I
Give students six strips of paper in six different colors. Repeat the Fraction Strip folding and labeling activity from the Exploring Fractions Task. This time, ask students to separate the Fraction Strips by cutting on the folds giving them 2 -1/2 strips, 3 –1/3 strips, and so forth. Give each student a plastic sandwich bag or envelope to store the strips. (You can also use fraction bars)
Arrange students in small groups of 2-3 students. Give them approximately ten minutes to write down their observations about the separated Fraction Strips. Have each group share some of their comments. Lead the groups to consider questions such as:

- Do you see any special relationships among the different colored strips?
- Place a ½ strip on your desk. How many strips or combinations of strips are the same size as ½?
- When fractions are the same size, they are called equivalent. What other equivalent sets of fractions can you create?

Have students line up their fraction strips and find as many relationships as they can. For instance, they might notice that three of the 1/6 pieces are equal to four of the 1/8 pieces, or that two of the 1/3 pieces are equal to four of the 1/6 pieces. Have students record these relationships on paper. When they have finished, have them share the relationships they have discovered. Record the relationships on chart paper and discuss.

1/6	1/6	1/6	1/6

1/3	1/3

Students will notice that one whole is the same as 2/2, 4/4, 8/8, 3/3, or 6/6. Another example includes the relationship between ½, 2/4, 4/8, and 3/6. Tell students that when fraction strips are the same length, they represent equivalent fractions. Students may also notice that for each of these fractions, the numerator is ½ of the denominator.

Part II
Students will work in small groups to answer the questions in the activity sheet. The teacher should monitor the groups, asking questions, and encouraging students to explore the concept of fractions.

Have groups (at least 2-3) share their solution to question numbers 6 and 7. Try to pick groups who presented different ways of solving the problems. After this lesson, have students store their Fraction Strips in a plastic sandwich bag.

Part III
Students can practice comparing fractions using the following activity adapted from Elementary and Middle School Mathematics: Teaching Developmentally by John A. Van de Walle, Karen S. Karp, and Jennifer M. Bay-Williams, p. 290.

The friends below are playing red light-green light. Who is winning? Use your fraction strips to determine how far each friend has moved.

Mary – 3/4	Harry – 1/2	Larry – 5/6
Sam – 5/8	Michael – 5/9	Angie – 2/3

Number Talk:
Strategy: Making Landmark or Friendly Numbers (Multiplication)
Often a multiplication problem can be made easier by changing one of the factors to a friendly or landmark number. Students who are comfortable multiplying by multiples of ten will often adjust factors to allow them to take advantage of this strength.

A common error students make when changing one of the factors to a landmark number is to forget to adjust the number of groups. The problem 9 x 25 can help us consider the common errors children make when making this adjustment. If 9 had been changed to 10, then the product of 250 would need to be adjusted not just by 1 but by one group of 25.

25 X 9 + 1 25 X 10 250 – 1 = 249 25 X 9 + 1 25 X 10 250 – 25 = 225	This common error arises when children are applying what works with addition and multiplication. They do not consider that they have changed the problem by adding on one group of 25 instead of a 1. When students understand that one group of 25 has been added, they will adjust the answer accordingly.

Below are two Making Landmark or Friendly Numbers Number Talks for you to try with your students.

2 x 25 4 x 25 6 x 25

4 x 5 4 x 10 4 x 50 4 x 49

For additional Number Talks using this strategy please see *Number Talks* by Sherry Parrish.

Common Misconceptions:
Students do not understand the importance of the whole of a fraction and identifying it. For example, students may use a fixed size of ¼ based on the manipulatives used or previous experience with a ruler.

Formative Assessment Questions:
- What relationships did you discover about fractions?
- How can you compare fractions?
- What equivalent groups of fractions did you discover?

Differentiation:
Extension
- Students can use coffee filters, paper plates, or other objects to create different models to illustrate inequalities.

Intervention
- Use ready-made Fraction Tiles or Virtual Manipulatives.

Vocabulary:
Relationship
Equivalent

Resources:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 3

Trash Can Basketball

Content Standard

3.NF.1. Understand a fraction 1/b (e.g., 1/4) as the quantity formed by 1 part when a whole is partitioned into b (e.g., 4) equal parts; understand a fraction a/b (e.g., 2/4) as the quantity formed by a (e.g., 2) parts of size 1/b. (e.g., 1/4)

3.NF.2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
a. Represent a fraction 1/b (e.g., 1/4) on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b (e.g., 4) equal parts. Recognize that each part has size 1/b (e.g., 1/4) and that the endpoint of the part based at 0 locates the number 1/b (e.g., 1/4) on the number line.
b. Represent a fraction a/b (e.g., 2/8) on a number line diagram or ruler by marking off a lengths 1/b (e.g., 1/8) from 0. Recognize that the resulting interval has size a/b (e.g., 2/8) and that its endpoint locates the number a/b (e.g., 2/8) on the number line.

3.NF.3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
a. Understand two fractions as equivalent if they are the same size (modeled) or the same point on a number line.
b. Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual fraction model).
c. Express and model whole numbers as fractions, and recognize and construct fractions that are equivalent to whole numbers. For example: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.
d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description
Students will play a game where they record a tally mark each time they shoot a trash ball into a trashcan. Students will write a fraction that represents the number of shots made and then create a poster that represents their results using an inequality.

Materials:

- "Trash Can Basketball" student recording sheet
- Each group will need 10 pieces of "trash" (paper balls).
- Box, tub, or trash can for a container
- Crayons or markers and construction paper for making a poster

****Please see link below for student recording sheet (pg. 90):**
http://tinyurl.com/MathTasks-Grade3-Unit5

Students collect data from playing "Trash Can Basketball." They use the data to write and compare fractions.
1. Students use scrap paper to make 10 paper balls per group. (Wad the paper balls up tightly so they are easier to aim.)
2. Place a trash can (or other large container) 5 feet away.
3. Students predict how many paper balls they will be able to get into the basket. Predictions should be written in the chart on the student recording sheet.
4. Students take turns with their partner(s) throwing the ten paper balls into the trash can. The partner will collect data using tally marks on the chart to show how many of the 10 paper balls went into the trash can.

The copy room is a good source of trash paper. Be sure the paper balls are tight. Loosely packed ones make it really difficult to throw accurately. Before beginning the throwing contest, as a class, decide on any rules regarding practice throws.

Number Talk:
Even though this task involves fraction standards, it is still important to practice number talks daily. There is an example of a number talk appropriate for 3rd grade below. For this number talk I focused on multiplication because fractions rely heavily on multiplication fluency. However, feel free to choose or create a number talk that is relevant and/or needed for the students in your classroom.

Strategy: Breaking Factors into Smaller Factors
Breaking factors into smaller factors instead of addends can be a very efficient and effective strategy for multiplication. The associative property is at the core of this strategy. It is a powerful mental strategy – especially when problems become larger and one of the factors can be changed to a one-digit multiplier.

12 x 25 (4 x 25) + (4 x 25) + (4 x 25) 100 + 100 + 100 = 300	Students will often approach a problem such as 12 x 25 by breaking the 12 into 3 groups of 4. They are comfortable with money amounts, and they will notice that four quarters are equal to one dollar.
(4 x 25) + (4 x 25) + (4 x 25) = 3 x (4 x 25) 12 x 25 = 3 x (4 x 25)	Help them connect their thinking to the associative property by recording the problem as 3 x (4 x 25). Encourage them to discuss whether 12 x 25 is the same as 3 x 4 x 25. This is one way to begin making a bridge into factors and using the associative property.
12 x 25 12 x (5 x 5) = (12 x 5) x5 60 x 5 = 300	We can also use the associative property and knowledge about factorization to think of 25 as 5 x 5.

Below are two Breaking Factors into Smaller Fraction Number Talks for you to try with your students.

2 x 3 x 4 4 x 3 x 2 6 x 4

2 x 3 x 8 4 x 2 x 6 6 x 8

For more Number Talks using this strategy please see *Number Talks* by Sherry Parrish.

Common Misconceptions:

Students do not understand the importance of the whole of a fraction and identifying it. For example, students may use a fixed size of ¼ based on the manipulatives used or previous experience with a ruler.

Formative Assessment Questions:

●How did you determine your score? How many times did you throw the paper ball? How many times did you "make a basket"?
● How did you compare your fraction to your opponent's?
● How did you determine your score? How many times did you throw the paper ball? How many times did you "make a basket"?
● How did you compare your fraction to your opponent's?

Differentiation:

Extension

● Repeat the activity as time permits. (Try different types of paper balls, distances, types of shots, etc.)

Intervention

● Have the chart pre-made on the poster for student use and/or allow student to write his/her results on a computer, print, and attach to the poster.

Vocabulary:

Compare
Predict/Prediction

Resources:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 3

Fill It Up!

Content Standard

3.MD.2. Estimate and measure liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm3 and finding the geometric volume of a container.)

Add, subtract, multiply, or divide to solve and create one-step word problems involving masses or volumes that are given in the same units (e.g., by using drawings, such as a beaker with a measurement scale, to represent the problem). (Excludes multiplicative comparison problems [problems involving notions of "times as much."])

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description

In this task, students estimate and compare liquid volume making connections to everyday items to build understanding of liquid volume and the liter.

Materials:

For each student:
- "Fill It Up" student recording sheet
- "Fill It Up, Measuring Stations" student recording sheet

For each group:
- a large pan or sheet of plastic (for spillage)
- a large graduated cylinder (1 liter)
- 2 different large containers (jar, bottle, bucket, pot, etc.)
- 1 bowl of water (may be colored for visual effect),
- 1 funnel

****Please see link below for recording sheets (pg. 73):**
http://tinyurl.com/MathTasks-Grade3-Unit6

Remember that the goal is for the students to develop a concept of liquid volume. According to Van de Walle (2006), "Children often confuse "holds more" with "taller" or "fatter," even though these may be misleading attributes. This is why a variety of container shapes not only adds interest but also can contribute to student understanding. (p.239)

Students will follow the directions below from the "Fill It Up" student recording sheet and the "Fill It Up Measuring Stations" recording sheet.

This task has four parts.
Part I: Introduction, Discussion, Connections
The teacher will facilitate a conversation with the students about liquid volume.
Discussion about the meaning of the concept and how it is measured should take place. The students and the teacher will make connections, cite examples, and clarify misconceptions. This would be a great opportunity to begin an anchor chart. The teacher should then show the class a container that measures one liter. Students should give examples of other containers that hold a liquid volume of about 1 liter to add to the anchor chart.

Part II: Exploring, Estimating, Comparing
In small groups, complete the mini activities below:
**Adapted from Teaching Student-Centered Mathematics, Van de Walle, Lovin, (2006)*

Capacity Sort

Provide a collection of labeled containers with one marked as the "target." The task is to sort the collection into those that hold more than, less than, or about the same as the "target" container. Provide a recording sheet on which each container is listed and a place to circle or write, "holds more," "holds less," and "holds about the same." List the choices twice for each container. The first choice is to record a guess made by observation. The second is to record what was found. (Beans, rice, liquid, or other fillers can be used to test estimates.)

Liquid Volume Line Up!

Given a series of five or six labeled containers of different sizes and shapes, order them from least capacity to most. Explain your thinking with your group members.
Notes for the teacher:
- Make sure that at least one of the containers measures 1 liter
- This can be quite a challenge, but let them "grapple" with it. Do not provide answers.
- Allow students to compare their findings with other groups.

Part III: Investigating, Estimating, Measuring
In small groups, students will explore, estimate, and measure liquid volume. Each group should have the following:
- a large plastic box or sheet of plastic (for spillage)
- a large graduated cylinder (1 liter)
- 2 different large containers (jar, bottle, bucket, pot, etc.)
- 1 bowl of water (may be colored for visual effect),
- 1 funnel

Using the funnel, have the students fill each of the containers until they believe that they have reached a liter. Once they have reached their estimate, allow them to pour the liquid from each container into the graduated cylinder. Ask the students to pay careful attention to what happens.

Part IV: Reflection
Once students have completed their work station task, ask students to complete the "Fill It Up" student recording sheet, and compare their estimations. Ask students to share their findings and to justify their findings by describing the process they followed.

Number Talk:
Even though this task involves measurement standards, it is still important to practice number talks daily. There is an example of a number talk appropriate for 3rd grade below. However, feel free to choose or create a number talk that is relevant and/or needed for the students in your classroom.

Strategy: Place Value and Negative Numbers
As students utilize this strategy, each number is broken apart into its respective place value. This can be represented by recording each number using expanded notation. Like place values are grouped and then subtracted. In problems where the standard U.S. regrouping algorithm could be used, negative values might result.

A. 123 – 59 (100 + 20 + 3) – (50 + 9) 100 | 20 | 3 - 50 | 9 100 – 30 – 6 100 – 30 = 70 70 – 6 = 64	Even though the student approaches the problem as individual columns of numbers, the value of each number is kept intact and used in the computation.
B. 123 – 59 123 - 59 100 (100 – 0) - 30 (20 – 50) - 6 (3 – 9) 64	Examples A and B demonstrate two different ways to record this strategy.
C. 999 - 345 600 (900 – 300) 50 (90 – 50) + 4 (9 – 5) 654	Example C models how this strategy would work if each part of the minuend (the whole) were a higher value than each part of the subtrahend (the parts being removed).

Below are two Place Value and Negative Numbers Number Talks for you to try with your students

5 – 5 5 – 6 5 – 7 5 – 8

20 – 10 2 – 4 22 – 14

For additional Number Talks using this strategy please see *Number Talks* by Sherry Parrish.

Common Misconceptions:
This is the students' first experience with measuring capacity. The terms and abbreviations, L and mL, may be confusing. Visual models made by the students will be important to display during this unit of study. Students may omit labels in final answers when recording answers.

Formative Assessment Questions:
- What is an efficient way to measure liquid capacity?
- When estimating liquid capacity, what do you need to consider?
- How much is a liter?
- What other containers have you seen in your everyday life with a capacity of one liter?
- Does the shape of the container change the amount of liquid it can hold? Why or why not?

Differentiation:

Extension
- Ask students to compare the relationships between the containers and the amount of liquid they can hold. Several things may come to light in this discussion.
    - The amount of liquid used to fill two containers can be the same, even though the shape of the containers may be different.
    - Having a benchmark to look at helps to make more accurate measurements.

Intervention
- Have an adult work with a small group of students who need support using a graduated cylinder.
- Have students complete the task using only one container.

Vocabulary:
Volume
Cylinder
Measure
Liquid Capacity
Liter

References:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Van de Walle, John A., and Lou Ann H. Lovin. Teaching Student-Centered Mathematics: Grades K-3, Volume 1. Pearson, 2006