Alaska Mathematics Standards
Math Tasks
Grade 4

Building 1,000

Content Standard

4.NBT.1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.
For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.

4.NBT.2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on the value of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description
This activity helps students begin to make some connections about place value, as well as gives them a visual perspective and sense of the number 1,000.

Materials:

- Building materials such as straws, toothpicks, noodles, string, pennies, paper clips, etc.
- Tape measures, rulers, yardsticks
- Poster board, markers, overheads for presenting, tape, glue, etc.

****Please see link below for "Building 1,000" question/recording sheet (pg. 26):**
http://tinyurl.com/MathTasks-Grade4-Unit1

Comments
This activity will allow the teacher to identify students who already have an understanding of 1,000 and how well they understand place value and number sense. This activity helps students begin to make some connections about place value, as well as gives them a visual perspective and sense of the number 1,000. The students get very excited thinking about what they might use to "build" 1,000 and are quite creative in their selection of materials. If used at the end, teachers will be able to see how well students understand place value and number sense, and use the knowledge of varying levels of student understanding to inform the use of the remaining tasks.

Task Directions
Students will answer the following questions on the student recording form:
- What does 1,000 look like? How long is it? How tall is it? How big is it? How much space will it take up?
- To answer these questions, decide what type of material you would like to use to show 1,000. Next, make a prediction about the size you think your 1,000 will be.
- Next, using words and pictures, explain what you did to make your prediction.

Once students have answered these questions, they will use materials they've chosen to create their model of 1,000. Then they will answer the following questions, and prepare their presentation for their classmates.
- What strategies did you use to create 1,000?
- What did you learn from this investigation of 1,000? Did you notice any patterns or connections?
- When you have completed this task, plan a presentation of your investigation for the class.

Number Talk:
Strategy: Breaking Each Number into Its Place Value
Once students begin to understand place value, this is one of the first strategies they utilize. Each addend is broken into expanded form and like place-value amounts are combined. When combining quantities, children typically work left to right because it maintains the magnitude of the numbers.

For example:

116 + 118	
(100 + 10 + 6) + (100 + 10 + 8)	Each addend is broken into its place value.
100 + 100 = 200	100's are combined.
10 + 10 = 20	10's are combined.
6 + 8 = 14	1's are combined.
200 + 20 + 14 = 234	Totals are added from the previous sums.

Below is a Breaking Each Number into Its Place Value Number Talk for you to try with your class:

28 + 11
14 + 35
22 + 15
18 + 31

15 + 27
23 + 18
17 + 25
16 + 27

For additional number talks using this strategy, please see *Number Talks* by Sherry Parrish.

Background Knowledge/Common Misconceptions:

This task helps assess students' previous knowledge and misconceptions about place value and number sense. The strategies they use to solve the problem demonstrate students' understanding about a number of concepts including place value, grouping, computation, number sense, patterns, and mathematical communication.

It is important that students understand that they are actually building the number 1,000. Have students brainstorm inexpensive and appropriate materials to build with. It is also a good idea to let parents know what you are doing and to ask them for donations. You will need to provide ample space for students to build and store their designs.

Formative Assessment Questions:

- How can you show the relationship between different place values?
- What do you know about 1,000 that you didn't know before?
- How can you show the different place values within 1,000?
- What makes 10 different from 100 and from 1000?
- What do you notice about how different materials work? Are some better than others at showing 1,000? Why or why not?
- Did you notice any patterns or connections?
- What strategies did you use to solve this?
- How can you effectively share what you've discovered?

Differentiation:

Extension

- Have students to experiment with larger numbers such as 10,000; 100,000; 100,000,000, etc.

Intervention

- Students may need to begin with 100.

Vocabulary:

Prediction
Place Value

References:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 4

Number Scramble

Content Standard

4.NBT.2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on the value of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description

In this task, students will manipulate the ten digits of the base ten-numeration system to complete various activities such as constructing large and small numbers and numbers with specific values in a given place, write numbers in expanded and standard form.

Materials:

- Scissors
- "Number Scramble" Recording Sheet
- Blank Place Value Chart

****Please see link below for recording sheet (pg. 30):**
http://tinyurl.com/MathTasks-Grade4-Unit1

In this task, students will manipulate the ten digits of the base ten-numeration system to complete various activities such as constructing large and small numbers and numbers with specific values in a given place, write numbers in expanded and standard form.

Comments

As students manipulate the numbers in this task, you will be able to see quickly which students have a good grasp of place value and the value of digits-of a number. For example, in steps 1 and 2, if students randomly place their numbers, they may need more practice to understand how the value of a number changes as its digits change.

Task Directions
Students will cut out number boxes (tiles) and use them to create numbers with the given requirements.
1. Make the largest whole number possible using 9 different tiles. Write your answer in standard form and expanded form.
2. Make the smallest whole number possible using 9 different tiles. Write your answer in standard form and expanded form.
3. Make a number worth more than two million, with a six in the ten-thousands place. Write the number in standard form and expanded form. Compare your number with your partner.
4. Make a number less than five million that has a two in the thousands' place. Write the number in standard form and expanded form. Compare your number with your partner.
5. Make a number that has only odd numbers in the thousands' period of the place value chart. Write the number in words.
6. Look carefully at your answers to Questions 1 and 2. Find one digit that is in
Both of your answers. How does the value of this digit change from the way you used it in Question 1 to the way you used it in Question 2? Use complete sentences to explain how and why the value of the digit did or did not change between the two answers.

Number Talk:
Strategy: Partial Products
This strategy is based on the distributive property and is the precursor for our standard U.S. algorithm for multiplication – it just keeps the place value intact. The strategy more closely resembles the algorithm when written vertically. When students understand that the factors in a multiplication problem can be decomposed or broken apart into addends, this allows them to use smaller problems to solve more difficult ones. As students invent Partial Product strategies, they can break one or both factors apart.

12 X 15			**12 X 15**		**12 X 15**	
Horizontal 12x15 12x (10+5) 12x10=120 (12x10) 12x5=60 (12x5) 120+60=180	Vertical 15 x 12 120 + 60 180	Whether the problem is written horizontally or vertically, the fidelity of place value is kept. In this example, the 15 is thought about as (10 + 5) while the 12 is kept whole.	(4 + 4 + 4) x 15 4 x 15 = 60 4 x 15 = 60 4 x 15 = 60 60 + 60 + 60 =180	This time the 12 is broken apart into (4 + 4 + 4) and the 25 is kept whole. The 12 could have been broken into (10 +2) or any other combination that would have made the problem accessible.	(10 + 2) x (10 + 5) 10 x 10 = 100 10 x 5 = 50 2 x 10 = 20 2 x 5 = 10 100+50+20+10 = 180	Both factors can be broken apart, and as numbers become larger, students often use this method until they become more confident in multiplying with larger quantities. It is difficult for some students to keep up with all of the parts of the problem, especially when trying to use this strategy without paper and pencil.

Below are two Partial Products Number Talks for you to try with your students.

2 x 7
4 x 7
4 x 8
3 x 8
8 x 7

3 x 26
6 x 26
9 x 26

For additional Number Talks using this strategy, please see *Number Talks* by Sherry Parrish.

Background Knowledge/Common Misconceptions:
Students should have had prior experiences and/or instruction with ordering, writing numbers in expanded and standard form and comparing large numbers.

Formative Assessment Questions:
- Explain how you decided the order of the digits.
- How can you tell which number is the largest or smallest?
- How does the value of a digit change when it is moved to the left on the place value chart? To the right?
- How could a place value chart help you if you are confused about which order the numbers.

Differentiation:

Extension
- Have students use all ten tiles to answer questions.
- Have students use tiles to create to develop two more additional questions to have their partners to solve.

Intervention
- Start students with building numbers in the hundreds, then the thousands, etc.
- Allow students to use a blank place value chart and write the numbers in the chart, showing the correct placement of the digits. This cueing device may assist students in comparing digits in the same place in order to determine value.
- Have students to use Base ten blocks to show their numbers.

Vocabulary:
Ones
Tens
Hundreds
Thousands
Millions
Place Value

References:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 4

Nice Numbers

Content Standard

4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

4.NBT.3. Use place value understanding to round multi-digit whole numbers to any place using a variety of estimation methods; be able to describe, compare, and contrast solutions.

4.NBT.4. Fluently add and subtract multi-digit whole numbers using any algorithm. Verify the reasonableness of the results.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description

In this task, students use estimates to add and subtract quantities.

Materials:

- Nice Numbers recording sheet
- Empty number lines
- Blank number lines labeled in different ways

****Please see link below for recording sheet (pg. 54):**
http://tinyurl.com/MathTasks-Grade4-Unit1

Comments:

For addition and subtraction problems involving only two terms, one strategy is to round only one of the two numbers. For example, you can round only the subtracted number as in 2367-1678 becomes 2367-1700. Rounding to "nice" numbers depends on what the estimator considers "nice". The point is that are no rigid rules. Choices depend on the relationships held by the estimator, on how quickly the estimate is needed, and how accurate an estimate needs to be.

Task directions:
Students will follow the directions below from the "Nice Numbers" recording sheet.
What is the approximate value of this coin collection? Justify your answer.

The most popular boy band is coming to town for a concert. The concert tickets cost $39.95. Parking at the arena cost $15. About how much will you pay to attend the concert? How do you know?

Robert and his family traveled from Atlanta, Georgia to Washington D.C. to visit the Martin Luther King Monument. They traveled a total 648 miles. It took them a total of 9 hours to get to Washington, D.C. If they traveled the same route back to Georgia, about how many miles would they drive? Explain your answer.

For the previous problem, determine the exact mileage for Robert's family's trip. Based on your estimation, is your answer reasonable? Explain.

Number Talk:
For this task I would explore the Multiplying Up Strategy. Using what students already know about place value and what happens to a number when they multiply it by 10, they can begin multiplying the divisor by 10, 20, 30 etc. until the find a "nice number" close to the dividend.

Strategy: Multiplying Up
Similar to the Adding Up strategy for subtraction, the Multiplying Up strategy provides access to division by building on the student's strength in multiplication. Students realize that they can also multiply up to reach the dividend. This is a natural progression as they become more confident in their use and understanding of multiplication and its relationship to division. Initially, students may rely on using smaller factors and multiples, which will result in more steps. This can provide an opportunity for discussions related to choosing efficient factors with which to multiply.

384 ÷ 16 10 x 16 = 160 10 x 16 = 160 2 x 16 = 32 2 x 16 = 32 10 + 10 + 2 + 2 = 24 24 x 16 = 384	This strategy allows students to build on multiplication problems that are comfortable and easy to use such as multiplying by tens and twos.

The open array can be used to model the student's strategy and link the operations of multiplication and division.

10	10	2	2	
16 x 10 = 160	16 x 10 = 160	16 x 2 = 32	16 x 2 = 32	16

Below are two Multiplying Up Number Talks for you to try with your students.

4 x 10
4 x 5
4 x 4
56 ÷ 4

2 x 10
2 x 5
2 x 2
38 ÷ 2

For additional Number Talks using this strategy please see *Number Talks* by Sherry Parrish.

Background Knowledge/Common Misconceptions:

According to Van De Walle, to round a number simply means to substitute a "nice" number that is close so that some computation can be done more easily. The close number can be any nice number and need not be a multiple of ten or one hundred. It should be whatever makes the computation or estimation easier or simplifies numbers sufficiently in a story, chart, or conversation. (*Van de Walle, Elementary and Middle School Mathematics, 2010*)

Formative Assessment Questions:

- What is the problem asking you?
- Does your answer make sense? How do you know?
- How does rounding help you in this context?

Differentiation:

Extension

- Plan a family trip, then estimate and calculate the mileage.

Intervention

- Provide students with a number line with a range of numbers noted.

Vocabulary:

Estimate
Round

Resources:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Van de Walle, John A., Karen S. Karp, Jennifer M. Bay-Williams. Elementary and Middle School Mathematics: Teaching Developmentally. Pearson, 2010

Alaska Mathematics Standards
Math Tasks
Grade 4

Investigating Prime and Composite Numbers

Content Standard	**Mathematical Practices**
4.OA.4. • Find all factor pairs for a whole number in the range 1–100. • Explain the correlation/differences between multiples and factors. • Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	**1. Make sense of problems and persevere in solving them.** **2. Reason abstractly and quantitatively.** 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets. 5. Use appropriate tools strategically. **6. Attend to precision.**

Task Description
Students will discover the difference between prime and composite numbers through the making of arrays using color tiles or counters.

Materials:

• Counters or color tiles

Students will discover the difference between prime and composite numbers through the making of arrays using color tiles or counters.

Task Directions
Have students create a T-chart and label one side only two ways and the other more than two ways. Instruct students to answer the following questions and fill in the T-chart with the answers to the questions.

How many ways can you make 2? Use your counters.
How many ways can you make 8? Use your counters.
How many ways can you make 9? Use your counters.
How many ways can you make 11? Use your counters.
How many ways can you make 24? Use your counters.
How many ways can you make 41? Use your counters.
How many ways can you make 15? Use your counters.
How many ways can you make 13? Use your counters.

Have a discussion with students about their observations of the number of arrays made for each number. Introduce the vocabulary words prime and composite.

Have students complete the activity with the following comparisons.

- Use your counters to determine if 21 is prime or composite. Explain your answer.
- Use your counters to determine if 14 is prime or composite. Explain your answer.
- Use your counters to determine if 7 is prime or composite. Explain your answer.

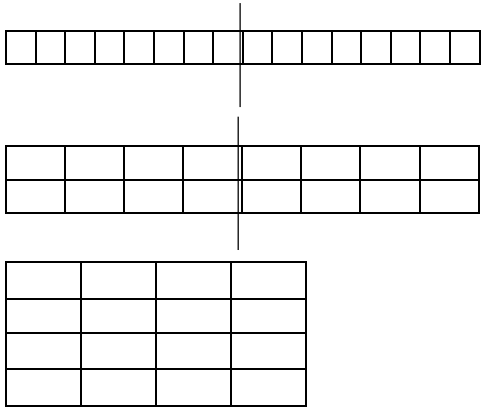
Use your counters to determine if 4 is prime or composite. Explain your answer.

Number Talk:

Strategy: Doubling and Halving

When students are provided opportunities to build arrays that have the same area and study the patterns of the dimensions, they often will notice a relationship that occurs between the factors or dimensions of the arrays. Consider the number 16. If we were to build all the possible arrays that would make 16 squares, we would have the following dimensions or factor pairs: 1x16, 2x8, 4x4, 8x2, 16x1

In every instance, we still have an area or product of 16, but our dimensions or factors have changed.

	When the 1x16 is halved, the number of rows doubles and the number of columns halve, resulting in 2 x 8. When the 2x8 is halved, the number of rows doubles and the number of columns halve, resulting in a 4x4. Doubling and halving can be continued until a 16x1 array is reached.

This strategy builds on the ease with which students double and halve numbers. We can apply this strategy to several problems.

8 x 25 /2 ( ) X2 4 x 50 /2 ( ) X2 2 x 100 = 200	The intent of the strategy is to change the problem into a friendly problem to solve. Once the student reaches a point where the solution is easily obtained, then he or she would not continue doubling and halving.	16 x 16 X2 ( ) /2 32 x 8 X2 ( ) /2 64 x 4 X2 ( ) /2 128 x 2 = 256	Doubling and halving is especially beneficial when multiplying with double digit problems. This can quickly turn the problem into a multiplication problem with a single-digit multiplier. **Note: some problems do not lend themselves to doubling and halving. This would be an important area for students to investigate.**

Below are two Doubling and Halving Number Talks for you to try with your students.

1 x 56
2 x 28
4 x 14
8 x 7

1 x 40
2 x 20
4 x 10
8 x 5

For more additional number talks using this strategy please see *Number Talks* by Sherry Parrish.

Background Knowledge/Common Misconceptions:

This standard requires students to demonstrate understanding of factors and multiples of whole numbers. This standard also refers to prime and composite numbers. Prime numbers have exactly two factors, the number one and their own number. For example, the number 17 has the factors of 1 and 17. Composite numbers have more than two factors. For example, 8 has the factors 1, 2, 4, and 8. A common misconception is that the number 1 is prime, when in fact; it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number.

Prime vs. Composite:

A prime number is a number greater than 1 that has only 2 factors, 1 and itself. Composite numbers have more than 2 factors. Students investigate whether numbers are prime or composite by building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g. 7 can be made into only 2 rectangles, 1 x 7 and 7 x 1, therefore it is a prime number) or by finding factors of the number.

Formative Assessment Questions:

- Are those all the ways you can make that number?
- What kind of number has only two ways it can be made?
- What kind of number has more than two ways it can be made?
- How do you know this number is prime? Composite?

Differentiation:

Extension

- Have students investigate using the counters to determine if 1 is prime or composite.
- Provide students with a list of numbers which are prime and a list of numbers which are composite. Have students prove the numbers are on the correct list by making arrays to determine the number of ways each number can be made.

Vocabulary:

Prime
Composite
Array

Resources:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 4

Sensible Rounding

Content Standard

4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

4.NBT.3. Use place value understanding to round multi-digit whole numbers to any place using a variety of estimation methods; be able to describe, compare, and contrast solutions.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description

This task provides several contexts in which students will have to determine the best estimation for the situation.

Materials:

- Sensible Rounding recording sheet
- calculator

****Please see link below for recording sheet (pg. 73):**
http://tinyurl.com/MathTasks-Grade4-Unit2

Task directions:
Students will follow the directions below from the "Sensible Rounding" recording sheet. Use an empty number line to work through the following problems. Discuss the answers carefully with your partner or group.

"Jane has 44 liters of milk to share among seven families. How much does she measure out for each family?" *(In traditional rounding "rules" students would round this amount to 7 liters. However, this context proves this rounded amount would be inaccurate because there is not enough milk for each family to receive 7 liters.)*

"The market gardener sends 44 tons of potatoes to eight supermarkets. How much does he send to each?" The market gardener would need x amount of tons of potatoes in order for each supermarket to receive 6 tons of potatoes. What amount does x represent? How do you know? (*Students will determine each supermarket will receive about 5 tons of potatoes with 4 tons left over for the gardeners. Students can use the remainder to help determine the value for x, or they can use the relationship of 6x8 to help them identify 48 as the value of x.)*

"The service station sells seven large pizzas for $44. About how much does a pizza cost?" Assume the service station charges to the nearest $1.

"John shares 44 Tootsie Pops among seven children. How many does each child get?" *(You cannot cut up Tootsie Pops, so 6r6 Tootsie pops ≈ 6 Tootsie pops. Have students discuss how many would be left over and who would get the extra Tootsie pops.)*

"Joel has 44 cookies to share among seven people. He needs to get rid of all the cookies. How many cookies does each person receive*?"(Here 6r6 is very inappropriate as an answer because he must get rid of all the cookies. Everyone receives six cookies. There are two whole cookies left over so probably these should be cut. Students need to decide if this amount can be considered 6 cookies or 7 cookies.)*

Number Talk:

Strategy: Partial Quotients

Like the Partial Products strategy for multiplication, this strategy maintains place value and mathematically correct information for students. It allows them to work their way toward the quotient by using friendly multipliers such as tens, fives and twos without having to immediately find the largest quotient. As the student chooses larger multipliers, the strategy becomes more efficient.

See below for example.

384/16

A.

```
      24
16 | 384
     -160      10
      224
     -160      10
       64              24
      - 32       2
        32
      - 32       2
         0
```

B.

```
      24
16 | 384
     -320      20
       64              24
      - 64       4
         0
```

When learning the procedure for the standard U.S. algorithm, students are often told that 16 cannot go into 3 (300), which is incorrect; 16 can divide into 3, but it would result in a fraction. With the Partial Quotients strategy, the "3" maintains its value of 300 and can certainly be divided by 16.

As the student works, he keeps track of the partial quotients by writing them to the side of the problem. When the problem is solved, the partial quotients are totaled and the final answer is written over the dividend.

Example A demonstrates using friendly 10s and 2s to solve the problem. As the 10s and 2s are recorded to the side of the problem, they represent 10 x 16 and 2 x 16.

Example B demonstrates a more efficient way to solve this problem.

Below are two Partial Quotients Number Talks for you to try with your students.

40/4
16/4
56/4

5/5
10/5
25/5
50/5
77/5

For additional number talks using this strategy please see *Number Talks* by Sherry Parrish.

Background Knowledge/Common Misconceptions:

A problem such as 44 ÷ 7 produces a calculator answer of 6.285714286 and the traditional division algorithm will produce 6r2. For any practical purposes, this number must be rounded. Yet there is no one rule for doing this. This is because the context always suggests the reason for rounding and the degree of precision required of the answer. For example, if the context is money, the degree of precision might be to the nearest hundredth (penny). The students need to display very good number sense and understanding of real problems to round calculator answers sensibly.

For students to be able to round accurately, "rounding should be flexible and well understood conceptually" (Van de Walle, 246). In order for students to conceptually understand rounding, they must be engaged in context to allow them to make sense of this concept. This task provides several contexts in which students will have to determine the best estimation for the situation. With these estimations, students will use the most familiar form of estimation, rounding (Van de Walle, 241).

Formative Assessment Questions:

- What is the problem asking you?
- Does your answer make sense? How do you know?
- How does rounding help you in this context?
- Did you get the same answer for 44 ÷ 7 each time you encounter it? Why or why not?

Differentiation:

Extension

- For each of these division problems, create word problems that are solved by the division yet the rounding rules change with the context. 2,225 ÷ 17; 4,567 ÷ 29; 7,888 ÷ 11 ...
- Have students use a calculator to divide and discuss how the results given by the calculator are not sensible answers. Students must explain a more sensible answer based on the context.

Intervention

- Allow students to use manipulatives to simulate the division contexts and discuss a sensible rounded answer.

Vocabulary:

Round/Rounding

Resources:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 4

Red Rectangles

Content Standard

4.NF.1. Explain why a fraction a/b is equivalent to a fraction (n × a)/ (n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2. Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½). Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description

For this activity students are asked to build several rectangles that are colored red over the same proportional area. (Adapted from EAT Cuisenaire's *Super Source Activity* entitled "Building Rectangles")

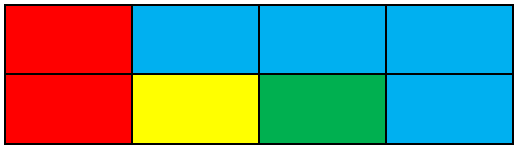
Materials:

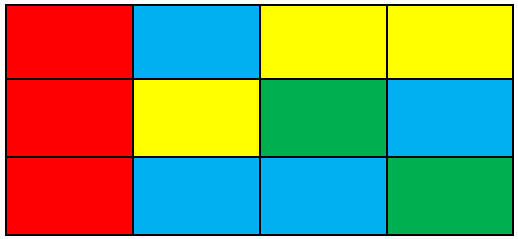
- Color Tiles
- "Red Rectangles" student recording sheet
- Die cut color tiles or pencils and crayons for recording findings

****Please see link below for recording sheet and graph paper printout (pg. 20):**
http://tinyurl.com/MathTasks-Grade4-Unit3

Comments:

For this activity students are asked to build several rectangles that are colored red over the same proportional area. However, each rectangle has to be a different, namely larger, size. Students will need a number or red color tiles but also they will need other colors as well. They will have to build several rectangles, they can do this with as much creativity as they choose, with the given proportions. For example, the following example can be utilized as a mini lesson or students could discover this on their own; If a child is asked to build a rectangle that is 1/4 red they could approach it several ways.





All three of these rectangles are a different size, however they all contain the same proportion of red tiles. Each is 1/4 red however a different number of tiles make up that proportion of 1/4. Students are then asked to write an equivalency statement similar to the one below, as well as record their work on graph paper:

1/4 = 2/8 = 3/12

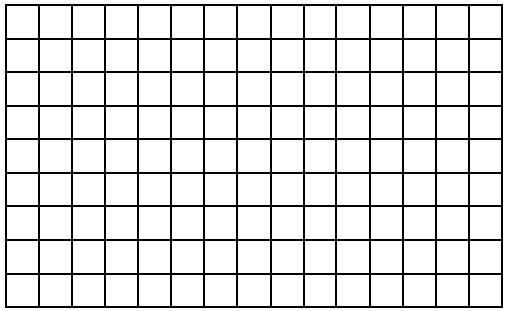
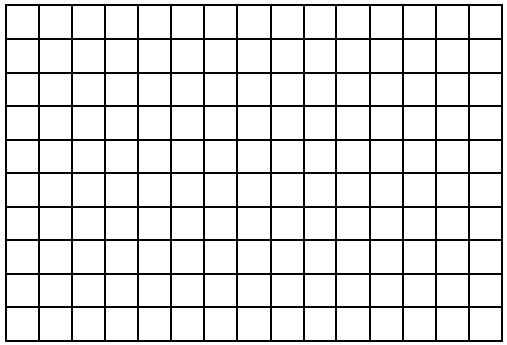
This is an excellent opportunity to discuss the importance of equivalency as it relates to the same-sized whole. These fractions are proportionally equivalent in relationship to these models, but they do not reference the same sized whole. Are they truly equal?

If available, students can glue die-cut color tiles. Additionally this task could be introduced by reading *The Hershey's Milk Chocolate Bar Fractions Book*, by Jerry Pallotta, illustrated by Rob Bolster or another story about equivalent fractions.

Task Directions:
How can you use Color Tiles to show fractional parts of different rectangles?
• Working with a partner, use Color Tiles to build more than 2 different-sized rectangles to represent each of these situations:
- $^{5}/_{6}$ of the tiles are red
- $^{2}/_{3}$ of the tiles are red
- $^{3}/_{4}$ of the tiles are red
- $^{2}/_{5}$ of the tiles are red

• Record your solutions. For each rectangle, write both the total number of tiles and the number of tiles that are red.
• Be ready to talk about how you know your solutions are correct.

<u>Number Talk:</u>
Strategy: Making Landmark or Friendly Numbers
Often multiplication problems can be made easier by changing one of the factors to a friendly or landmark number. Students who are comfortable multiplying by multiples of ten will often adjust factors to allow them to take advantage of this strength.

9 X 15 + 1 (group of 15) 10 X 15 = 150 150 – 15 = 135 15  **9** 15  **10**	With this strategy, notice that not just one, but one group of 15 was added. This is a very important distinction for students and one that comes as they develop multiplicative reasoning. Since one extra group of 15 was added, it now must be subtracted. The initial problem was 9 X 15, but it was changed to 10 X 15, which resulted in an area of 150 squares. The extra group of 15 is subtracted from the total area to represent the product for 9 X 15.

Below are two Making Landmark or Friendly Numbers, Number Talks for you to try with your students.

2 X 25 4 X 25 6 X 25	5 X 5 5 X 10 5 X 30 5 X 29

For additional Number Talks using this strategy please see *Number Talks* by Sherry Parrish.

Background Knowledge/Common Misconceptions:

Encourage students to organize their thinking to be sure they have found as many ways as possible to represent equivalent fractions. Possible equivalent fractions are shown below. Before asking students to work on this task, be sure students are able to:

- identify the number of equal pieces needed cover ⅔ of a rectangle or array
- show equivalent fractions with colored tiles
- record on the student sheet equivalent fractions either by coloring or gluing die cut color tiles
- write an equation which shows the equivalent fractions

Formative Assessment Questions:

- How are you keeping your work organized?
- Have you found all more than 2 possible equivalent fractions? How do you know?
- How do you know these two fractions are equivalent?
- How many different illustrations can be created to show equivalent fractions? How do you know?
- How did you go about deciding how many tiles should be in each rectangle?
- How did you figure out how many red tiles to use?
- How did you find a second solution for each situation?
- How do the numbers in the fraction relate to the different tiles you used to create your rectangles?
- Look at the posted solutions. Are there any that you think are not correct? If so, tell why.
- Do you think that there are other solutions beyond those posted? Explain.
- How is it possible that there could be more than one solution for each situation?

Differentiation:

Extension

- Students could explore denominators other than those listed in the standard, such as 9, 15, 24, etc.
- Students could experiment with only two colors and begin to make discoveries about what the remaining fraction may be. For example, if 1/4 of our tiles are red the remaining blue tiles must be 3/4.
- Students could also make very large rectangles of up to 100 and begin to discover some of the equivalent fraction that work with decimal fraction nicely such as 1/4, 1/5, 1/10, 1/2, etc.
- Students could continue far beyond 3 or more rectangles and challenge one another to find as many as possible, which could lead to a discussion of whether there are a finite number of ways to represent a fraction.

Intervention

- Allow students to begin by representing rectangles that are ½ red prior to exploring other fractions.
- Some students may be more successful utilizing virtual manipulatives like those found at http://nlvm.usu.edu/en/nav/frames_asid_203_g_2_t_1.html?from=category_g_2_t_1.html

Vocabulary:

Rectangle	Equivalent Fraction
Fractional Part	Solution

Resources:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 4

Equivalent Fractions

Content Standard

4.NF.1. Explain why a fraction a/b is equivalent to a fraction (n × a)/ (n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2. Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½). Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description

This task allows students to explore the relationship between equivalent fractions and write equations for equivalent fractions using the product and quotient of a fraction equivalent to one.

Materials:

- "Equivalent Fractions, 2/3" student recording sheet
- "Equivalent Fractions, 3/4" student recording sheet

****Please see link below for both recording sheets (pg. 61):**
http://tinyurl.com/MathTasks-Grade4-Unit3

Comments:

This task allows students to explore the relationship between equivalent fractions and write equations for equivalent fractions using the product and quotient of a fraction equivalent to one.

This task is adapted from an activity called "Slicing Squares" on p. 311 in *Elementary and Middle School Mathematics: Teaching Developmentally*, By John Van De Walle (2007). See the section on "Equivalent-Fraction Concepts."

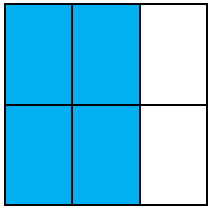
Give students the opportunity to explore equivalent fractions with this task. Encourage students to look for patterns, both in the models as well as in the numerical representations. Equivalent fractions can be thought of as different names for a fraction.

Once students have written equivalent fractions and are able to show that the fraction was multiplied by a fraction equivalent to 1, then begin the discussion about using the inverse operation. Ask students how they can simplify a fraction by dividing it by a fraction equivalent to 1. See the examples below.

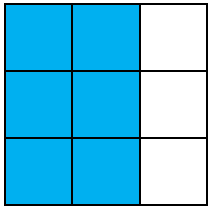
$2/3 = 2/3 \times 4/4 = 8/12$; $8/12 = 8/12 \div 4/4 = 2/3$

$3/4 = 3/4 \times 2/2 = 6/8$; $6/8 = 6/8 \div 2/2 = 3/4$

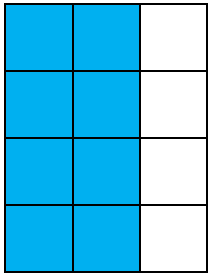
Possible solutions to the "Equivalent Fractions 2/3" student recording sheet are shown below:



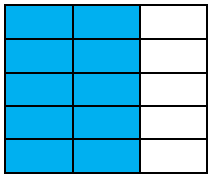
2/3 x 2/2 = 4/6



2/3 x 3/3 = 6/9

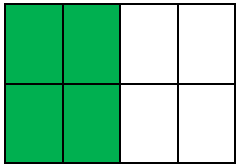


2/3 x 4/4 = 8/12

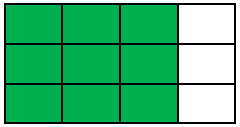


2/3 x 5/5 = 10/15

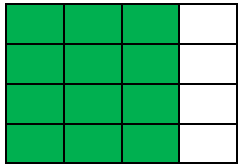
Possible solutions to the "Equivalent Fractions 3/4" student recording sheet are shown below.



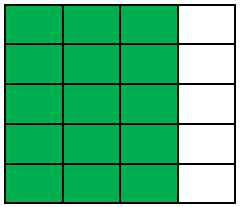
3/4 x 2/2 = 6/8



3/4 x 3/3 = 9/12



3/4 x 4/4 = 12/16



3/4 x 5/5 = 15/20

Task Directions

Students will follow the directions below from the "Equivalent Fractions – 2/3" student recording sheet.

Find fractions that are equivalent to the fraction shown in each square below. Slice the squares by drawing horizontal line segments in each square to create a different but equivalent fraction. Then write an equation for each model. See the example below.

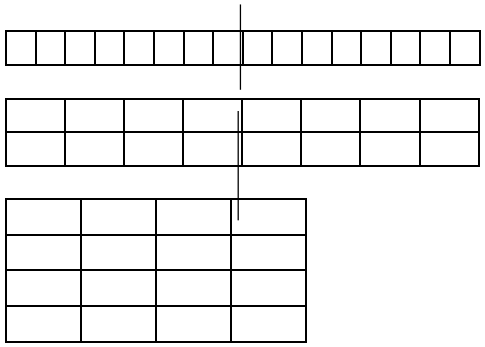
Students will follow the directions below from the "Equivalent Fractions –3/4" student recording sheet. Find fractions that are equivalent to the fraction shown in each square below. Slice the squares by drawing horizontal line segments in each square to create a different but equivalent fraction. Then write an equation for each square.

Number Talk:

Strategy: Doubling and Halving

When students are provided opportunities to build arrays that have the same area and study the patterns of the dimensions, they often will notice a relationship that occurs between the factors or dimensions of the arrays. Consider the number 16. If we were to build all the possible arrays that would make 16 squares, we would have the following dimensions or factor pairs: 1x16, 2x8, 4x4, 8x2, 16x1

In every instance, we still have an area or product of 16, but our dimensions or factors have changed.



When the 1x16 is halved, the number of rows doubles and the number of columns halve, resulting in 2 x 8.

When the 2x8 is halved, the number of rows doubles and the number of columns halve, resulting in a 4x4.

Doubling and halving can be continued until a 16x1 array is reached.

This strategy builds on the ease with which students double and halve numbers. We can apply this strategy to several problems.

8 x 25 /2 ( ) X2 4 x 50 /2 ( ) X2 2 x 100 = 200	The intent of the strategy is to change the problem into a friendly problem to solve. Once the student reaches a point where the solution is easily obtained, then he or she would not continue doubling and halving.	16 x 16 X2 ( ) /2 32 x 8 X2 ( ) /2 64 x 4 X2 ( ) /2 128 x 2 = 256	Doubling and halving is especially beneficial when multiplying with double digit problems. This can quickly turn the problem into a multiplication problem with a single-digit multiplier.
Note: some problems do not lend themselves to doubling and halving. This would be an important area for students to investigate.			

Below are two Doubling and Halving Number Talks for you to try with your students.

8 X 16 4 X 32 2 X 64

1 X 32 2 X 16 4 X 8 8 X 4 16 X 2 32 X 1

For additional number talks using this strategy please see *Number Talks* by Sherry Parrish.

Background Knowledge/Common Misconceptions:
Constructing the idea that fractions are relations and that the size or amount is relative to the whole is a critical step in understanding fractions. Fair sharing contexts also provide learners with opportunities to explore how fractional parts can be equivalent without necessarily being congruent. They may look different but still be the same amount. Students have worked with the concept of fair share or partitioning since 2nd grade, with standards which refer to same-sized shares or equal shares. Students should have knowledge of vocabulary terms such as: *numerator and denominator*.

Some common misconceptions, found in *Math Misconceptions*, that children have include:
● Dividing nontraditional shapes into thirds, such as triangles, is the same as dividing a rectangle into thirds. If they are only used to dividing traditional shapes – circles, squares, and rectangles –they begin to think that all shapes are divided similarly.
● Children often do not recognize groups of objects as a whole unit. Instead they will incorrectly identify the objects. For example, there may be 2 cars and 4 trucks in a set of 6 vehicles. The student may mistakenly confuse the set of cars as 2/4 of the unit instead of 2/6 or 1/3 (Bamberger, Oberdorf, & Schultz-Ferrell, 2010).

Therefore, it is important that students are exposed to multiple units of measure, various shapes, and denominators other than halves, thirds, and fourths. Additionally, the denominator used as an expression of the whole is a key concept to express for mastery.

Formative Assessment Questions:
● Into how many parts did you slice each piece?
● What is a fraction that is equivalent to one? (If the student sliced each piece into three parts, they need to multiply the fraction by 3/3.)
● How could you use equivalent fractions to simplify this fraction (i.e.6/9)?

Differentiation:
Extension
● Invite students to play the Fraction Game.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=18
This game requires students to recognize equivalent fractions. Students should think about scenarios from the game that could be presented to the class so that students can discuss strategies and choices available to them.

Intervention
● Some students may benefit from having a table of equivalent fractions. Ask students to complete the table by multiplying the numerator and denominator by the same number.

Vocabulary:
Numerator
Denominator
Equivalent Fraction

References:
Bamberger, Honi J., Christine Oberdorg, Karren Schultz-Ferrell. Math Misconceptions, PreK-Grade 5: From Misunderstanding to Deep Understanding. Portsmouth: Heinemann, 2010

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Van de Walle, John A., Karen S. Karp, Jennifer M. Bay-Williams. Elementary and Middle School Mathematics: Teaching Developmentally. Pearson, 2007

Alaska Mathematics Standards
Math Tasks
Grade 4

Write About Fractions

Content Standard

4.NF.1. Explain why a fraction a/b is equivalent to a fraction (n × a)/ (n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2. Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½). Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description

In this task, students use understanding of equivalent fractions compare fractions with different denominators. (Adapted from Fosnot, C. *The Fieldtrip, Context for Learning Mathematics.*)

Materials:

- Connecting cubes
- Strips of equal length paper
- Write About Fractions task sheet

****Please see link below for task sheet (pg. 71):**
http://tinyurl.com/MathTasks-Grade4-Unit3

Comments:

Students have had some opportunities to work with comparing unit fractions with different denominators in context. By exploring this procedure in context and examining the results, students are supported to develop deep understandings about fractions, thereby avoiding common misconceptions.

Task directions:

Students will follow the directions below from the "Write About Fractions" task sheet. Write a convincing argument for the following statement:

1/2 + 2/3 does not equal 2/5

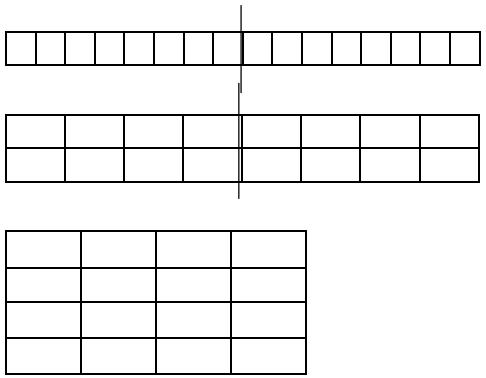
Number Talk:

Strategy: Doubling and Halving

When students are provided opportunities to build arrays that have the same area and study the patterns of the dimensions, they often will notice a relationship that occurs between the factors or dimensions of the arrays. Consider the number 16. If we were to build all the possible arrays that would make 16 squares, we would have the following dimensions or factor pairs:

1x16, 2x8, 4x4, 8x2, 16x1

In every instance, we still have an area or product of 16, but our dimensions or factors have changed.

	When the 1x16 is halved, the number of rows doubles and the number of columns halve, resulting in 2 x 8. When the 2x8 is halved, the number of rows doubles and the number of columns halve, resulting in a 4x4. Doubling and halving can be continued until a 16x1 array is reached.

This strategy builds on the ease with which students double and halve numbers. We can apply this strategy to several problems.

8 x 25 /2 ( ) X2 4 x 50 /2 ( ) X2 2 x 100 = 200	The intent of the strategy is to change the problem into a friendly problem to solve. Once the student reaches a point where the solution is easily obtained, then he or she would not continue doubling and halving.
16 x 16 X2 ( ) /2 32 x 8 X2 ( ) /2 64 x 4 X2 ( ) /2 128 x 2 = 256	Doubling and halving is especially beneficial when multiplying with double digit problems. This can quickly turn the problem into a multiplication problem with a single-digit multiplier. **Note: some problems do not lend themselves to doubling and halving. This would be an important area for students to investigate.**

Below are two Doubling and Halving Number Talks for you to try with your students.

1 X 56
2 X 28
4 X 14
8 X 7

1 X 48
2 X 24
4 X 12
8 X 6
16 X 3

For additional number talks using this strategy, please see *Number Talks* by Sherry Parrish.

Background Knowledge/Common Misconceptions:

Many students develop misconceptions about adding fractions, thinking the numerators can be added and denominators can be added. For example, they might think that 1/2 +1/3 is equivalent to 2/5.

Formative Assessment Questions:

- What do you notice about the unit fractions you have created?
- How can drawing a model help you answer this question?
- What strategies can you use to help write about this topic?
- Can you give me an estimated size of the fraction you created with that cut?
- Does your answer make sense? How do you know?

Differentiation:

Extension

- In addition to this task, student can identify a fraction that is equal to ½ + ⅓.

Intervention

- Provide fraction strips the students can manipulate to determine their response.
- Encourage students to use words, numbers and/or pictures in their explanation.

Vocabulary:

Unit Fraction
Numerator
Denominator
Equivalent Fraction
Estimated

References:

Fosnot, Catherine Twomey. Contexts for Learning Mathematics. Portsmouth: Heinemann, 2007

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 4

Pizza Party

Content Standard

4.NF.3. Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions (e.g., by using a visual fraction model).
Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1+ 1 + 1/8 = 8/8 + 8/8 + 1/8.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description
In this individual task, students will draw fraction representations and use those representations to add and subtract fractions.

Materials:

- "Pizza Party" student recording sheet
- "Pizza Party, Pizza Dough" student sheet (each sheet has enough circles for two students)
- Colored pencils or crayons
- Glue stick

****Please see link below for recording sheet and pizza dough student sheet (pg. 18):**
http://tinyurl.com/MathTasks-Grade4-Unit4

Using fraction models divided into eighths (pizzas), students create addition and subtraction story problems.

Comments
One way to introduce the task is by describing a family tradition of having pizza and a movie every Friday evening. Explain that the family makes two pizzas for dinner and rents a movie for the family to watch. There is always one cheese pizza and one pepperoni pizza. Each pizza is cut into eight equal slices.

Discuss with the students some possible addition problems that could be done with the pieces of pizza. For example, if the mom ate two slices of cheese pizza and one slice of pepperoni pizza, how much pizza did she eat?

Discuss the whole is cut into 8 equal pieces, so 2/8 cheese + 1/8 pepperoni = 3/8 of a pizza. Have a student record the number sentence on the board, reminding students about the correct fraction notation.

As a subtraction problem, one example would be discussing the amount of cheese pizza left after the mom took two pieces. 8/8 –2/8 = 6/8. Ask students how they might illustrate subtraction with the pizza slices. (Students may suggest crossing out the pieces removed or circling the pieces that are being subtracted.)

Sometimes students find it difficult to understand that the whole can be any shape. Therefore, it may be helpful to provide square pizzas for students to work with in addition to the circle-shaped pizzas used in this task.

Time does not permit all students to share their work with the class. However, students may be afforded the opportunity to share their work in a small group and then one student from each group may share with the whole group. Or students can share their work with a partner and two or three students can be selected to share their work with the class. Teachers need to be thoughtful about who will share during the closing of a lesson. The student(s) whose work is shared needs to add to the class discussion or take the class discussion in a specific direction. A teacher needs to think about what type of conversation will help clarify possible student misconceptions and solidify student understanding of the concepts imbedded in the task.

Task Directions
Students will follow the directions below from the "Pizza Party" student recording sheet.

You will be writing two story problems, modeling the problems using pizzas that you create. Fold this paper in half to create two sections on the back to record your stories.
1. Create two pizzas.
a. Cut out two circles of paper (pizza dough) and color them to look like your two favorite types of pizza.
b. Fold the pizzas into eighths.
2. Fold this paper in half to create two sections on the back to record your pizza stories.
3. Write an addition story problem on the back of this paper.
a. Cut out the correct number of pizza slices for your story.
b. Glue down the pizza slices to illustrate your story.
c. Explain how you solved the problem using words and numbers.
4. Write a subtraction story problem on the back of this paper.
a. Cut out the correct number of pizza slices for your story.
b. Glue down the pizza slices to illustrate your story.
c. Explain how you solved the problem using words and numbers.
Be prepared to share your story, illustration, and solution with the class.

Number Talk:
Strategy: Making Landmark or Friendly Numbers (Multiplication)
Often a multiplication problem can be made easier by changing one of the factors to a friendly or landmark number. Students who are comfortable multiplying by multiples of ten will often adjust factors to allow them to take advantage of this strength.

A common error students make when changing one of the factors to a landmark number is to forget to adjust the number of groups. The problem 9 x 25 can help us consider the common errors children make when making this adjustment. If 9 had been changed to 10, then the product of 250 would need to be adjusted not just by 1 but by one group of 25.

25 x 9 + 1 25 x 9 + 1	25 x 10 250 – 1 = 249 25 x 10 250 – 25 = 225	This common error arises when children are applying what works with addition and multiplication. They do not consider that they have changed the problem by adding on one group of 25 instead of a 1. When students understand that one group of 25 has been added, they will adjust the answer accordingly.

Below are two Making Landmark or Friendly Numbers Number Talks for you to try with your students.

2 x 25 4 x 25 6 x 25	4 x 5 4 x 10 4 x 50 4 x 49

For additional Number Talks using this strategy, please see *Number Talks* by Sherry Parrish.

Background Knowledge/Common Misconceptions:

Students think that it does not matter which model to use when finding the sum or difference of fractions. They may represent one fraction with a rectangle and the other fraction with a circle. They need to know that the models need to represent the same whole. In addition students have a tendency to add both the numerator and denominator when using different operations with fractions as opposed to just adding numerators.

The students should have had multiple opportunities with paper-folding fractions. To create eighths, students can fold the pizza in half, then in fourths, and finally into eighths. Each student's story problems may be unique. To assess student work, look for an illustration made with the pizza slices that matches the events in the story, an accurate number sentence for the story, and clear explanations. Student explanations should provide evidence that they understood why the denominator is 8. The standard explicitly says students should write their fractions as the sum of 1/b. Guide students toward this goal, having them write number sentences that reflect this. For example, if someone ate ⅜ of a pizza then they actually ate one slice, then another, then a third slice or ⅛ + ⅛ + ⅛. You could simply joke around with kids about how no one really stuffs three slices in their mouth at once!

Formative Assessment Questions:

- In your addition story, how many pieces of pizza do you have in all? How many slices of pizza in one whole? How do you write that as a fraction?
- In your subtraction story, how many pieces of pizza do you have left? How many slices of pizza in one whole? How do you write that as a fraction?
- Why does the denominator stay the same with addition and subtraction?
- Tell me the story that goes with your picture and number sentence.

Differentiation:

Extension

● Have students consider the whole to be both pizzas, for a total of 16 slices equaling one whole.

● What would happen if the pizza restaurant made a mistake and cut one of the pizzas into fourths? How does it make finding the answer to an addition or subtraction sentence more difficult if the denominators of your fractions are not the same? Have students write problems where one pizza is cut into fourths, the other is cut into eighths.

Intervention

● Allow students to tell their story and model it with their pieces in a small group before gluing and labeling it on paper.

● If students have more experience with fraction tiles that are already labeled with unit fractions than they can manipulate them to determine their sums and differences.

● Students may also benefit from having the blank circles the place their decorated pizzas on top of, this visual may help them see what part is missing or serve as a reminder as to what the whole represents. Students can think of the blank circles as the pizza pan.

● Some students may benefit from adding or subtracting their pizzas slices on a number line.

Vocabulary:

Numerator
Denominator
Number Sentence

References:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 4

Fraction Clues

Content Standard

4.NF.3. Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions (e.g., by using a visual fraction model).
Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1+ 1 + 1/8 = 8/8 + 8/8 + 1/8.
c. Add and subtract mixed numbers with like denominators (e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction).

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description

In addition to determining the fraction of a set, and determining the set when given a fraction, students will now have to also find equivalent fractions.

Materials:

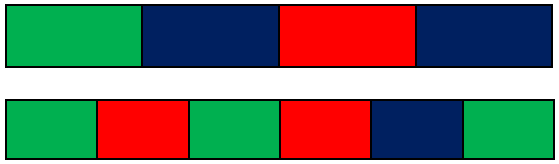
- Colored tiles
- Fraction Clues recording sheet
- Crayons or colored pencils

****Please see link below for recording sheet (pg. 118):**
http://tinyurl.com/MathTasks-Grade4-Unit4

In this task students will use what they have learned about adding and subtracting fractions, using equivalent fractions and multiplying a fraction by a whole number to give another student clues about the fraction strip they created. There is a lot of emphasis on communicating mathematically in this task.

Comments

To introduce this activity display these two fraction bars made from Color Tiles.



Ask students to find out what portion of the whole a tile in the first bar represents and what portion of the whole a tile in the second bar represents. Students should be able to determine that each tile in the first bar represents 1/4 of the whole and each tile in the second bar represents 1/6 of the whole.

Ask students to explain what fractional part each color represents in each fraction bar. Give the following set of fraction clues that describe one of the fraction bars. Stop after each clue and ask children which fraction bar is the solution and how they know.
● The fraction bar is one-half green
● The fraction bar is one-third red
● The fraction is one-sixth blue
Many children will not need all three clues to determine the solution however they should be comfortable arguing and verifying their answers and they may need all three clues to conclude that the solution is the second bar.

Part 2 is a much more challenging version where students create fraction bars with any number of tiles, requiring students to use different denominators, such as 6, 8, 10, and 12. This allows students to develop other strategies for determining the denominator, for example a student may be forced to find a common denominator or they may figure out on their own that the largest denominator must refer to the total number of tiles.

If available, students can glue die-cut red, yellow, blue and green squares.

Task Directions
Students will follow directions below from the Fraction Clues activity sheet.
● Obtain a set of colored tiles.
● Work with a partner to make a fraction bar and record it on their activity sheet.
● Write at least 3 clues that describe your fraction bar
● Exchange only your clues with another group
● Represent your answer with a number sentences (for example: if you have 10 tiles and 1/2 are red then write the number sentence 10/2 = 10 x 1/2 which is 5 tiles)
● Attempt to build another group's fraction bar as they attempt to build yours.
● Discuss results with each other.

Number Talk:
Several number talk strategies can assist students in finding equivalent fractions. Repeated addition (multiplication) and subtraction (division) are strategies that many students may choose to use.

Strategy: Division: Repeated Subtraction or Sharing/Dealing Out
In this strategy, the student associates the divisor with the number of groups between which the whole is being shared. Possible ways to make this explicit are suggested in the example that follows using the problem 12/2:

- If students share their strategy as 12 -2, -2, -2, -2, -2, -2
- Scaffold to multiplication with 3 x 2 = 6, 3 x 2 = 6
  So…6 x 2 = 12
  So…12/2 = 6

Repeated Subtraction affords an excellent vehicle for discussing efficiency: Is it more efficient to subtract 2s or to multiply a group of 2? Is there a way we can build on something we know, such as 3 x 2, to make this problem more efficient? Each situation offers opportunities to help students think flexibly, fluently, and efficiently.

Below is a Repeated Subtraction or Sharing/Dealing Out Number Talks for you to try with your students.

10/2 14/2 25/5

For additional number talks and information about this strategy, please see *Number Talks* by Sherry Parrish.

Background Knowledge/Common Misconceptions:

Students think that it does not matter which model to use when finding the sum or difference of fractions. They may represent one fraction with a rectangle and the other fraction with a circle. They need to know that the models need to represent the same whole. In addition students have a tendency to add both the numerator and denominator when using different operations with fractions as opposed to just adding numerators.

Students need practice with open-ended activities that allow them to design their own problems and then assess one another. This activity also makes students use mathematical language, verify answers, and work collaboratively with another student. This activity offers students a concrete way to see equivalent fractions. This activity also helps build the "guess and check" strategy as each student tries to build the fraction bar based on the set of clues.

This activity is also valuable because students start to realize that a different number of tiles in a different fraction bar can still be represented by the same fraction.

For example:

3 tiles

4 tiles

In the first bar three yellow tiles represent 1/2 and in the second bar four tiles represent 1/2. Students will gain further understanding that the number of tiles being used (numerator) is always dependent on its relationship to the total number of tiles (denominator). Before asking students to work on this task, be sure students are able to:

- identify the number of equal pieces needed to cover one whole as the denominator
- show equivalent fractions with an area model
- record on the student sheet equivalent fractions or fraction sets (either by coloring or gluing die cut squares)
- write an equation which shows the clues and verify their answer.

Formative Assessment Questions:

- What clues did you write to describe your fraction bar?
- Have you found all of the possible equivalent fractions? How do you know?
- Were you able to build the fraction bar based on the clues? If not, why?
- Could you change any of your clues?
- What number sentence can describe the tiles in your bar?

Differentiation:

Extension

● Once students have completed the task above, this lesson can be extended to have two pairs of students combine their fraction bars to make a larger fraction bar, then continue the activity writing clues for another group to solve.

● Students could also be encourage to work with larger fraction bars as well as write more clues for determining those fraction bars. Most color tiles only have red, blue, green and yellow tiles, so the activity will never have more than four fractions to represent.

● Often the clue with the largest denominator tells you how many tiles can be used. However, students could be challenged to use only 2 clues and therefore force them into situations where they need to find common denominators. For example my fractions are 1/4 red and 1/3 green. They will then need to build several bars that have 12 or 24 tiles.

Intervention

● If necessary students could begin this activity with a smaller set, such as using only four tiles.

● If students are struggling, they could attempt with activity with only three colors instead of using all four colored tiles.

Vocabulary:

Equivalent Fraction
Numerator
Denominator
Unit Fraction
Fraction Bar
Number Sentence

References:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 4

Measuring Up Decimals

Content Standard

4.NF.5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.

4.NF.6. Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

4.NF.7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual model).

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description
Students will use the metric system to represent decimals.

Materials:

- Meter sticks
- Objects of varying lengths (including those larger than 1 meter) labeled with a letter

Comments
This is a constructing task, so it is important that as students begin to write the decimal representations of the objects they measure that they have the opportunity to struggle and consider what the "whole" is in this situation. As they join together lengths in the latter part of the task, allow them to use their own methods for adding the tenths and hundredths together. DO NOT show them the place value addition model, as they need to develop this method on their own. Fourth grade students should focus on using the idea of decimals fractions to combine decimals.

Measurement discussion will be sure to come out during this task. Use this lesson as an opportunity to build background for the upcoming Measurement Unit. Choose a variety of objects to measure, including objects over 1 meter in length so that students must problem-solve to measure and write the decimal length.

Task:
Review the various models of decimals and reintroduce the meter as one model for showing linear decimals. Introduce today's task with students focusing on the 3 parts they are to complete.

Students will follow the directions below.

You have a variety of objects to measure today!

● Measure each object and write its length in centimeters and meters. (You may need to write the length in meters as a decimal of a meter.)
● Choose 3 pairs of object and combine their lengths. Write the combination length in terms of meters. Explain how you combined the lengths, using fraction and decimal notation.
● Choose 3 pairs of objects to compare their lengths. Use a model to explain how you compared the lengths of the objects.

After the students complete the task, have each pair or group share their work. Focus their discussion on:

- The methods they used for representing the objects that measured less than a meter as a decimal of a meter
- The methods they used for combining the lengths of two objects and the mathematical representations they used for this
- The methods they used for comparing the lengths of the objects and the visual models they used to defend their thinking

● After and while groups are sharing, have them look for groups that had efficient strategies, the similarities between the methods used, and the differences.

Number Talk:

****This number talk is a great introduction to this task. Students will practice adding numbers by breaking them into place value. This same strategy can be used with decimal numbers. However, do not use this strategy with decimal numbers until students have worked through the task first!**

Strategy: Breaking Each Number into Its Place Value

Once students begin to understand place value, this is one of the first strategies they utilize. Each addend is broken into expanded form and like place-value amounts are combined. When combining quantities, children typically work left to right because it maintains the magnitude of the numbers.

For example:

116 + 118 (100 + 10 + 6) + (100 + 10 + 8)	Each addend is broken into its place value.
100 + 100 = 200	100's are combined.
10 + 10 = 20	10's are combined.
6 + 8 = 14	1's are combined.
200 + 20 + 14 = 234	Totals are added from the previous sums.

Below are two Breaking Each Number into Its Place Value Number Talks for you to try with your class:

28 + 11
14 + 35
22 + 15
18 + 31

15 + 27
23 + 18
17 + 25
16 + 27

For additional number talks using this strategy, please see *Number Talks* by Sherry Parrish.

Background Knowledge/Common Misconceptions:

Students treat decimals as whole numbers when making comparison of two decimals. They think the longer the number, the greater the value. For example, they think that .03 is greater than 0.3.Furthermore, students do not understand how the places in decimal notation have the same correspondence (places to the left are 10 times greater than the places to their immediate right) as the places in whole numbers.

The meter stick is a length model of decimals that is a familiar context for the use of decimals. This lesson provides students a visual model for seeing the centimeters within a meter as decimals of a whole meter, as well as seeing each decimeter as a tenth of the meter stick.

As students share their strategies and methods for writing the decimal representations, comparing the decimals, and combining lengths of objects, make sure they are explaining their thinking and critiquing the reasoning of others.

The task directions purposely give few directions on "how" student must complete this task. You may wish to brainstorm ahead of time ways to organize their work (tables and charts) and important information for communicating their ideas (number sentences using the comparison symbols, using grid models were needed, etc.), but it is important that students have the opportunity to show their thinking and each step in a way that makes sense to them. The discussion you have with students during the task completion should focus on guided questions rather than guiding statements. For instance, asking, "What do you know about the relationship between centimeters and meters?" when students are writing the length of an object in terms of meters, rather than saying, "Don't forget that there are 100 centimeters in a meter and a meter is the whole" puts the responsibility and opportunity for thinking back on the student.

Formative Assessment Questions:

- How did you know how to write the length of an object shorter than a meter in terms of a meter?
- When you combined the lengths of objects, how did that change the decimal representations you used?
- How did decimals help you compare the lengths of objects? What models of decimals helped you prove your comparisons?
- Were students able to see the connection between tenths and hundredths as regrouping to the next unit higher?
- How did students show connections between tenths and hundredths?

Differentiation:

Extension

- Have students also write the lengths of the objects in terms of decimeters. Have a discussion comparing the lengths written in centimeters, decimeters, and meters. How do these measurements look different? What causes the decimal to "move" places?

Intervention

- If the measurement of the objects is an issue, label the objects ahead of times in terms of centimeters so that they focus can be on the decimal representation of that length in terms of a meter.
- Have students complete a decimals grid for each length to use for comparing and combining the lengths of the objects.

Vocabulary:

Decimal	Tenths
Length	Hundredths
Meter	Compare

Resources:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 4

Taxi Trouble

Content Standard

4.NF.5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.

4.NF.6. Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

4.NF.7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual model).

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description
Students will add tenths and hundredths and compare decimals.

Materials:

- Paper
- Pencils
- "Taxi Trouble" Student Sheet

****Please see link below for student sheet (pg. 122):**
http://tinyurl.com/MathTasks-Grade4-Unit5

Task:
- Introduce the problem. Make sure students understand they are to defend their choice and use mathematics (shown in number and word form) to defend their choices.
- Have students briefly read the task and make predictions about which Taxi Company they think will be the best deal. Have them explain their thinking for their predictions.

Students will follow the directions below from the "Taxi Trouble" recording sheet.

Sam is in downtown Atlanta and needs to take a taxi 5 miles to the convention center. There is a sign posted with the different taxi companies and their rate.

Taxi Company A: $4.00 sitting fee and 30/100 of a dollar for every 1/10 of a mile.
Taxi Company B: Free sitting fee and 5/10 of a dollar for every 1/10 of a mile.
Taxi Company C: $10.00 sitting fee and 2/10 of a dollar for every 1/10. (Sam has a 1/10 off of your total price coupon)

Which Taxi cab company should Sam choose to ride to the convention center?

Number Talk:

Strategy: Breaking Each Number into Its Place Value

Once students begin to understand place value, this is one of the first strategies they utilize. Each addend is broken into expanded form and like place-value amounts are combined. When combining quantities, children typically work left to right because it maintains the magnitude of the numbers.

For example:

116 + 118 (100 + 10 + 6) + (100 + 10 + 8)	Each addend is broken into its place value.
100 + 100 = 200	100's are combined.
10 + 10 = 20	10's are combined.
6 + 8 = 14	1's are combined.
200 + 20 + 14 = 234	Totals are added from the previous sums.

Below are two Breaking Each Number into Its Place Value Number Talks for you to try with your class:

354 + 111
267 + 232
215 + 136
342 + 64

135 + 219
315 + 192
167 + 173
115 + 293

For additional number talks using this strategy, please see *Number Talks* by Sherry Parrish.

Background Knowledge/Common Misconceptions:

Students treat decimals as whole numbers when making comparison of two decimals. They think the longer the number, the greater the value. For example, they think that .03 is greater than 0.3. Furthermore, students do not understand how the places in decimal notation have the same correspondence (places to the left are 10 times greater than the places to their immediate right) as the places in whole numbers.

Students may need some background knowledge built on how taxi companies charge for their services. Many of them have a flat fee plus an additional rate per mile or fraction of a mile traveled. Often the flat fee is a distractor from the per mile rate. It is important that students make predictions from their initial reading of the rate and then compare that with the actual result. This will show them how important it is to do the math when making choices on how to spend their money!

Formative Assessment Questions:

Have each pair or group share their work. Focus their discussion on:

- How are you determining the cost of the ride for each Taxi Company?
- How are you organizing your work?
- Where have you used decimal fractions and decimal to defend your thinking?
- Which company they thought was best
- The mathematical justification for their thinking
    - The methods they used for determining the cost of each company
    - How they combined the tenths and hundredths
- After and while groups are sharing, have them look for groups that had efficient strategies, the similarities between the methods used, and the differences between the methods used.
- Which strategies for combining tenths and hundredths did you see today that worked best?
- Where you surprised by the results?
- What did you learn about the decimal representations of the money being spent?
- Were students able to find the correct price for each company using decimals and decimal fractions?
- How did students show connections between tenths and hundredths?

Differentiation:

Extension

- Have students create their own taxi company and write its sitting fee and charge per mile in terms of tenths of a mile. Have them compare their company's price with the companies listed.

Intervention

- Have students use grids, money manipulatives, and/or other concrete models to build each amount of money for the ride. Use this concrete model as the basis for the number representations they use to explain their thinking.

Vocabulary:

Tenths
Hundredths
Decimal
Convention Center

Resources:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 4

Angle Shape Sort

Content Standard	**Mathematical Practices**
4.G.1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular, parallel, and intersecting line segments. Identify these in two-dimensional (plane) figures.	**1. Make sense of problems and persevere in solving them.** 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets. **5. Use appropriate tools strategically.** **6. Attend to precision.** 7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting. 8. Look for and express regularity in repeated reasoning.

Task Description
In this task, students will sort shapes by angles.

Materials:

- 3 bendable straws/Wikki Sticks/Pipe Cleaners per student
- Paper shape cutouts
- Angle sorting student task sheet

****Please see link below for paper shape cutout template and student task sheet (pg. 22):**
http://tinyurl.com/MathTasks-Grade4-Unit6

Part I
Tell students that today you will learn about something called angles. Remind students that an angle is formed when two lines or sides share a vertex. Show students several angles on the board. Ask students to look for angles throughout the room. After students have found several angles, tell students that there are three types of angles that we will discuss this year: acute, obtuse, and right.

Show students how you can create angles through different parts of your body, like your arms or your ankles. Show students a 90 degree angle with your ankle. Tell students that this is called a right angle. Next, show them an acute angle by pulling your toes up toward your shin. Last, show them an obtuse angle by pointing your toes and stretching them away from your shin. Allow the students to try showing the angles with their ankles as you say the words "right angle", acute angle", or "obtuse angle". You can also do this with your arms. Have them make a strong bicep "muscle" to demonstrate a right angle. Then draw your fist closer to your shoulder to create an acute angle and extend your forearm moving the fist away from the shoulder to create an obtuse angle. Ask the students if the length of their

foot or leg changes the size of the angle. How about the length of the arm? Why or why not? Talk with the students about the fact that an angle represents the size of the opening between your foot and leg or your upper and lower arm.

Part II
Review the three types of angles with students. Give each student three bendable straws, Wikki-Sticks, or pipe cleaners. Have students use the material to form each type of angle (acute, obtuse, or right). Have them show their angles to a partner to check. Then, give each set of partners a set of sticks (coffee stirrers etc.) and ask them to play "pick up" sticks. Students will gather a fist full of straws and then carefully drop them from a kneeling position. Once all sticks have dropped, they should locate angles. The teacher should circulate and ask students to identify angles they found. This game time should only last a few minutes.

Part III
Give each student a sorting sheet and shape handout. Have students cut out each of the shapes. Then, give each student two coffee stirrers/straws/Wikki-sticks/pipe cleaners. Students can measure one straw using the corner of their paper and tape it at a 90 degree angle. Students can then manipulate the other straw to match the angles of each shape. Another option is to use an index card to locate a right angle. Next, they can compare the manipulated straw to the right angle straw to determine if the angle is right, obtuse, or acute. After measuring, encourage students to draw the shape in the correct section of the chart.

While students are working, ask questions like:
- What shape are you working with? How did you know its name?
- How many angles does your shape have?
- What types of angles does your shape have? How did you figure that out?
- Where will you place your shape on the chart?
- Did you have to use the straws each time? If not, how did you determine what the angle was?

Part IV
Have students come together to share the placement of each of the shapes. The teacher should prepare larger versions of each shape and the sorting sheet. Allow partner groups to place the shapes in the correct sections. Students should justify the placement of each shape by explaining their strategies for determining the types of angles. Encourage the audience to ask questions and make comments about the placement of the shapes.

Number Talk:
Even though this task involves a geometry standard, it is still important to practice number talks daily. There is an example of a number talk appropriate for 4th grade below. However, feel free to choose or create a number talk that is relevant and/or needed for the students in your classroom.

Strategy: Keeping a Constant Difference (Subtraction):
As students begin to understand subtraction as the difference between two quantities, they can investigate what occurs if both numbers are changed by the same amount. Allowing students to explore this relationship with smaller problems such as 5-3 is a way to help them build this understanding. If 5 and 3 are both changed by +2, the problem 7-5 will result. Notice there is still a difference of 2. What if we removed 2 from each number in the problem 5-3? We would then create the problem 3-1, which still results in a difference of 2. Adding or subtracting the same quantity from both the subtrahend and minuend maintains the difference between the numbers. Manipulating the numbers in this way allows the student to create a friendlier problem without compromising the result.

123 - 59 123 + 1 = 124 - 59 + 1 = - 60 64	Both numbers have been adjusted by +1, which makes a problem with an easy 10. Deciding on the amount to subtract or add to adjust the problem is a big decision. For instance, would it have been helpful to adjust each number by -1? This would have created the problem 122 – 58, which is not an easier problem to solve.

Below are two Keeping a Constant Difference Number Talks for you to try with your students.

14 – 10
13 – 9
14 – 7
15 - 6

61 – 29
62 – 30
59 – 27
49 – 17

For more Number Talks using this strategy, please see *Number Talks* by Sherry Parrish.

Background Knowledge/Common Misconceptions:

Students believe a wide angle with short sides may seem smaller than a narrow angle with long sides. Students can compare two angles by tracing one and placing it over the other. Students will then realize that the length of the sides does not determine whether one angle is larger or smaller than another angle. The measure of the angle does not change.

Students should have had prior experiences and/or instruction with plane figures and angles. A common misconception that many students have is that wide angles with short sides may seem smaller than a narrow angle with long sides. Students can compare two angles by tracing one and placing it over another. Students will then realize that the length of the sides does not determine whether one angle is larger or smaller than another angle. The measure of the angle is not dependent on the lengths of the legs.

Formative Assessment Questions:

- Could students distinguish between the three types of angles?
- Were students able to determine the types of angles in each shape?
- Could students explain and justify their thinking as they sorted the shapes by types of angles?

Differentiation:

Extension

- Ask the students to write descriptors for a bingo style game using large student task sheet from this task.
- Students can take the angle hunt task sheet around school for a scavenger hunt. Challenge them to find various angles.

Intervention

- Play a bingo style game with different variations of task sheet.
- Partner students together for an Angle Hunt scavenger hunt around the school.

Vocabulary:

Angle
Acute
Obtuse
Right Angle
Vertex

Resources:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 4

Quadrilateral Roundup

Content Standard

4.G.1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular, parallel, and intersecting line segments. Identify these in two-dimensional (plane) figures.

4.G.2. Classify two-dimensional (plane) figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description
During this task, students will investigate and explain properties of quadrilaterals.

Materials:

For Each Group:
- Three pieces of yarn or three plastics hoops
- A set of "Quadrilateral Pieces" for each group of students
- Labels for each group from "Labels"
- Blank index cards
- Markers
- Measuring tools such as rulers and index cards for students to test for right angles

****Please see link below for quadrilateral pieces, labels and student task sheet (pg. 53):**
http://tinyurl.com/MathTasks-Grade4-Unit6

Students will be using Venn diagrams to classify figures, so it is advisable to review Venn diagrams with students beforehand by modeling a sort, such as those quadrilateral pieces having no right angles and those having at least 1 right angle.

The purpose of this task is for students to become familiar with the properties of quadrilaterals and their defining characteristics as a context for classifying figures by the absence or presence of angles of a specified size and/or parallel and perpendicular lines. This task is meant to elicit discussion about not only the size of the angles in each type of quadrilateral, but the types of lines used to make the sides. While students may sort the quadrilateral pieces in many ways, keep in mind that the focus is on the types of angles and the types of lines used to make the sides of the quadrilaterals.

Some properties of quadrilaterals that may be discussed are included below. As students draw conclusions about the relationships between different figures, be sure they are able to explain their thinking and defend their conclusions. Much of the information below may come out as a result of students' explorations. This is information to look for and highlight as they explore the quadrilaterals, not a list of understandings that you must teach them beforehand.

- A shape is a quadrilateral when it has exactly 4 sides and is a polygon. (To be a polygon the figure must be a closed plane figure with at least three straight sides.)
- A rectangle is a parallelogram with 4 right angles and 2 sets of parallel sides.
- A square is a rectangle with sides of equal length.
- A parallelogram is a quadrilateral with 2 sets of parallel sides.
- A rhombus is a parallelogram with sides of equal length.

Task Directions

PART I

The students will place all 16 quadrilateral pieces in a Venn diagram they create from pieces of string or three hoops. They will use the labels from the "Label" sheet to direct their sorts. Students may leave shapes outside of the rings. Encourage them to think of a label that could be placed for the entire group if there was one big circle around both rings and the ones that fall outside of the rings. The same set of pieces can be used for several sorts using the different labels and/or several sets can be recreated so that students can glue their sorts onto mats or posters for sharing. During the sorting, circulate among groups and ask students to explain and defend their placement of the figures in the different rings. After each sort use the following questions to guide discussion.

- Why did you place any shapes at all in the intersection there? What characteristics does it have?
- What do all the shapes on one ring have in common? The other?
- How much are the shapes in the ring different?
- What different label would eliminate one or more shapes form the ring?
- What different label for the one of the rings would allow you to include a new shape?

PART II

Give students the "Unknown Labels" figures to reverse this investigation. On this sheet, students are given the pre-sorted shapes in rings and then asked to determine which label could go above each ring. Students must then use the properties of the shapes (angles and parallel or perpendicular lines) to defend their labels.
Possible Solutions for "Unknown Labels"
Set 1: At least one pair of parallel sides (left), no side parallel (right)
Set 2: All sides the same length (inner), At least one pair of parallel sides (outer)
Set 3: At least one obtuse angle (left), At least one right angle (right).

Number Talk:

Even though this task involves a geometry standard, it is still important to practice number talks daily. There is an example of a number talk appropriate for 4th grade below. However, feel free to choose or create a number talk that is relevant and/or needed for the students in your classroom.

Strategy: Breaking Factors into Smaller Factors

Breaking factors into smaller factors instead of addends can be a very efficient and effective strategy for multiplication. The associative property is at the core of this strategy. It is a powerful mental strategy – especially when problems become larger and one of the factors can be changed to a one-digit multiplier.

12 x 25 (4 x 25) + (4 x 25) + (4 x 25) 100 + 100 + 100 = 300	Students will often approach a problem such as 12 x 25 by breaking the 12 into 3 groups of 4. They are comfortable with money amounts, and they will notice that four quarters are equal to one dollar.
(4 x 25) + (4 x 25) + (4 x 25) = 3 x (4 x 25) 12 x 25 = 3 x (4 x 25)	Help them connect their thinking to the associative property by recording the problem as 3 x (4 x 25). Encourage them to discuss whether 12 x 25 is the same as 3 x 4 x 25. This is one way to begin making a bridge into factors and using the associative property.
12 x 25 12 x (5 x 5) = (12 x 5) x5 60 x 5 = 300	We can also use the associative property and knowledge about factorization to think of 25 as 5 x 5.

Below are two Breaking Factors into Smaller Fraction Number Talks for you to try with your students.

2 x 3 x 4 4 x 3 x 2 6 x 4	2 x 3 x 8 4 x 2 x 6 6 x 8

For more Number Talks using this strategy, please see *Number Talks* by Sherry Parrish.

Background Knowledge/Common Misconceptions:

Students believe a wide angle with short sides may seem smaller than a narrow angle with long sides. Students can compare two angles by tracing one and placing it over the other. Students will then realize that the length of the sides does not determine whether one angle is larger or smaller than another angle. The measure of the angle does not change.

Students should have the following background knowledge.

- Be able to use a straight edge or ruler to draw a straight line.
- Know how to use a ruler, and how to identify right angles (90 degrees), obtuse angles, and acute angles (using the corner of an index card or another object with a known angle of 90 degrees).
- Understand that the side across from an angle on a triangle can be described as an opposite side
- Know parallel means that lines will never intersect or cross over each other no matter how long they are extended.
- Understand that perpendicular means lines or segments intersect or cross forming a right angle. (Some students may use a known 90 degree angle to show an angle is a right angle.)
- Know that a property is an attribute of a shape that is always going to be true. It describes the shape.
- Be able to use a ruler to measure sides to verify they are the same length.

Formative Assessment Questions:

- Why did you place any shapes at all in the intersection there? What characteristics does it have?
- What do all the shapes on one ring have in common? The other?
- How much are the shapes in the ring different?

● What different label would eliminate one or more shapes form the ring?
● What different label for one of the rings would allow you to include a new shape?
● How can you be sure that label for the Unknown group is correct? What if your proof?
● Were students able to use the presence or absence of certain angles to classify the figures?

Differentiation:

Extension

● Students can create their own label and challenge a partner to sort the shape using their labels.
● Students can create their own "Unknown Labels" samples for other students to label.

Intervention

● Have students label each shape with its known properties (perpendicular lines, 1 right angle), etc. and use those as an aid when sorting.

Vocabulary:

Quadrilateral
Rectangle
Parallelogram
Parallel
Perpendicular
Right Angle
Obtuse Angle
Acute Angle

Resources:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010

Alaska Mathematics Standards
Math Tasks
Grade 4

What's The Story?

Content Standard

4.MD.1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two column table.
For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36).

4.MD.2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

4.MD.4. Solve real-world problems involving elapsed time between U.S. time zones (including Alaska Standard time) (L)

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics. Students draw pictures using dot cards, number lines, picture cards, and counters to represent and compare quantities or sets.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure. Students will use tally marks to represent benchmarks (5, 10) of counting.
8. Look for and express regularity in repeated reasoning.

Task Description
In this task students will make a line plot to display data to 1/8 inch.

Materials:

- Set of data that is teacher or student generated or "What's the Story" Recording Sheet (one per student)
- Markers, colored pencils, or crayons
- Graph paper or chart paper
- Ruler

****Please see link below for "What's the Story" student recording sheets (pg. 26):**
http://tinyurl.com/MathTasks-Grade4-Unit7

In this task, students will be working with graphs and data sets. Given a set of data, students will create a graph, describe a context for the data, explain a possible collection method, and report what they learn from the data. The set of data used can be student or teacher created.

Comments

You may want to demonstrate this type of activity as a whole class before assigning this task. The students should have graph paper or chart paper available in case they choose to use it. Students should also have the opportunity to share their solutions. The set of data can be determined by the teacher so that the data set can apply to different learning levels, and degrees of difficulty. The teacher also has the option of allowing students to create their own data set.
A sample set of data could be as follows: 1/8, 1/8, 2/8, 2/8, 3/8, 3/8, 3/8, 3/8, 3/8, 4/8, 4/8, 5/8, 5/8, 5/8, 5/8, 6/8, 6/8, 7/8, 7/8, 7/8, 7/8, 8/8, 8/8, 8/8, and 8/8.
For the sample data given, there are many possible situations students may come up with. The following are some sample stories for the given data:

- We sampled sets of 8 m&m's checking for the number of red in each set. The first sample we took had 7 reds out of 8 m&m's or 7/8, the second sample had only 3 out 8 m&m's red or 3/8 . . .
- I measured my sunflower plant every three days to check its growth. After the first 3 days (the first sample), I noticed it grew 7/8 of an inch. After the second 3 days (the second sample), I noticed it grew 3/8 of an inch . . .

Task Directions

Have students follow the directions below:
Use your set of data to:

- Display the data on a line plot
- label your line plot appropriately
- create a situation that would fit the set of data given
- explain how the set of data was/might have been collected
- give at least five real-world interpretations from the given set of data on your Recording Sheet

Number Talk:

Strategy: Multiplying Up

Similar to the Adding Up strategy for subtraction, the Multiplying Up strategy provides access to division by building on the student's strength in multiplication. Students realize that they can also multiply up to reach the dividend. This is a natural progression as they become more confident in their use and understanding of multiplication and its relationship to division. Initially, students may rely on using smaller factors and multiples, which will result in more steps. This can provide an opportunity for discussions related to choosing efficient factors with which to multiply.

384 ÷ 16 10 x 16 = 160 10 x 16 = 160 2 x 16 = 32 2 x 16 = 32 10 + 10 + 2 + 2 = 24 24 x 16 = 384	This strategy allows students to build on multiplication problems that are comfortable and easy to use such as multiplying by tens and twos.	The open array can be used to model the student's strategy and link the operations of multiplication and division. 10 | 10 | 2 | 2 16 x 10 = 160 | 16 x 10 = 160 | 16 x 2 = 32 | 16 x 2 = 32 16

Below are two Multiplying Up Number Talks for you to try with your students.

6 X 10
6 X 5
6 X 6
6 X 2
99/6

3 X 10
3 X 20
3 X 3
3 X 2
68/3

For additional Number Talks using this strategy, please see *Number Talks* by Sherry Parrish.

Background Knowledge/Common Misconceptions:

Students need a good understanding of the different ways to represent data in a graph in order to choose the most appropriate graph for the data presented. The data presented above are most appropriate for a bar graph or circle graph. Data for a line graph would need to be given as an ordered pair or with two pieces of information (in a t-chart for example) for each data point. Line graphs are frequently used to display data over time.

Be sure students understand that the bars in bar graphs should not be attached to one another. A small space must be placed between each bar within the graph. Histograms are similar to bar graphs in that they use bars, but represent continuous data; therefore they do not have spaces between each bar. (This will be discussed in sixth grade.)

Know how to create a bar graph.

Students use whole-number names when counting fractional parts on a number line. The fraction name should be used instead. For example, if two-fourths is represented on the line plot three times, then there would be six-fourths.

Specific strategies may include:

Create number lines with the same denominator without using the equivalent form of a fraction. For example, on a number line using eighths use 48 instead of 12. This will help students later when they are adding or subtracting fractions with unlike denominators. When representations have unlike denominators, students ignore the denominators and add the numerators only. Have students create stories to solve addition or subtraction problems with fractions to use with student created fraction bars/strips.

Formative Assessment Questions:

- How did you decide what kind of situation would be appropriate to describe the data in your line plot?
- What are ways in which these data could have been collected?
- Would the data be appropriate on another type of graph? If so which graph(s)?
- Is there another way that your data could have been collected?
- Are there other interpretations you can make from your line plot?
- Have you labeled your line plot appropriately?

Differentiation:

Extension

- Have students repeat the activity using data that they collect.
- Have students display their data in an appropriate graph.
- Have students research and describe situations in which data are collected and displayed routinely.

Intervention

- Have students work with a smaller data set for the task.
- Allow students to collect or create data for their project.

Vocabulary:

Line plot Data

Resources:

Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5. Sausalito: Math Solutions Publications, 2010