Kindergarten Mathematics Standards

Comparison Tool for Standards Transition

Updated March 2012

This document can be used to assist educators in analyzing the commonalities and differences between the new Alaska Mathematics standards and the Fourth Edition (Grade Level Expectations). This document is a first start toward a transition and districts may choose to augment with more detail.

The first column contains the new math standards. The second column shows the Grade Level Expectations (GLEs) that align to the new standards. The third column provides comments, usually highlighting differences between the new standards and GLEs that align in higher grades. Additionally, the comments may include a notation about an increase in rigor. Rigor may be defined as a standard that requires deeper understanding, higher order thinking, expanded analytical processes, or simply a skill introduced at an earlier grade.

Note that some GLEs are coded with an L. This signifies that the GLE was not assessed on the statewide assessment; it was to be assessed at the local level. No new standards are identified as being for local assessment. Students advancing through the grades are expected to meet each year’s grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.

In most cases there are not complete matches between the two sets of standards, and it should not be assumed that either the content or skills found in one set of standards will match completely with those of the other set.

| **New Math Standards** | **Grade Level Expectations** | **Comment** |
| --- | --- | --- |
| 6.RP.2. Understand the concept of a unit rate (*a*/*b* associated with a ratio *a:b* with *b ≠*0, and use rate language in the context of a ratio relationship) and apply it to solve real world problems (e.g., unit pricing, constant speed).  *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”* | **[6] E&C-5** developing and interpreting scale models  Any aligned GLE found in the higher grades will need to be absorbed in the lower grade as part of the transition. | Grade 6 GLE provides a specific real world model for understanding unit rate.  **[7] E&C-6** solving proportions using a given scale  **[8] E&C-5** using ratio and proportion |

The new standards represent a shift in the purpose of the standards. They are more instructional in nature, intended to guide classroom curriculum. The new standards do not serve as an assessment document unlike the GLEs. The Department with the support of stakeholders will prepare an assessment framework which will guide the development of the new assessments. The new standards will be assessed starting spring 2016. Until then, all districts will continue administering the Standards Based Assessments aligned to the GLEs through spring 2015.

A table at the end shows the GLEs not matched to the new standards. The comment column indicates where the GLE may be matched to a new standard in a lower or higher grade. Although some GLEs will be taught at other grade levels, teachers must provide opportunities for these GLEs to be reviewed in preparation for the spring Standards Based Assessments through spring 2015.

| **Grade 6 Math GLEs not matched by new standards** | **Comments** |
| --- | --- |
| **The student demonstrates conceptual understanding of fractions (proper or mixed numbers), decimals, percents (whole number), or integers by**  **[6] N-2** identifying place value positions from thousandths to millions (L) (M1.2.2) | 4th and 5th Grade Standards  **(4.NF.6, 4.NF.7, 5.NBT.3)** |

This GLE must be reviewed prior to the SBA through spring 2015.

Finally, the new standards for each grade level define what students should understand and be able to do by the end of each grade. They correspond to Standards of Mathematical Practice. The Standards of Mathematical Practice describe characteristics and traits that mathematics educators at all levels should seek to develop in their students. They describe ways that students should be engaging with mathematics as they progress through school. The integration of these standards into classroom instruction is a key strategy for increasing cognitive demand and conceptual learning. The Standards for mathematical Practice are included at the end of the document.

**Kindergarten Overview**

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| **Counting and Cardinality (CC)**  • Know number names and the count sequence.  • Count to tell the number of objects.  • Compare numbers.  **Operations and Algebraic Thinking (OA)**  • Understand addition as putting together and  adding to, and understand subtraction as  taking apart and taking from.  **Number and Operations in Base Ten (NBT)**  • Work with numbers 11–19 to gain foundations  for place value.  **Measurement and Data (MD)**  • Describe and compare measurable attributes.  • Classify objects and count the number of  objects in categories.  **Geometry (G)**  • Identify and describe shapes.  • Analyze, compare, create, and compose  shapes. | **In Kindergarten, instructional time should focus on two critical areas:**  (1) representing, relating, and operating on whole numbers, initially with sets of objects;  (2) describing shapes and space.  More learning time in Kindergarten should be devoted to number than to other topics. |
| **Mathematical Practices (MP)**   1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning. |

**Alaska New Mathematics Standards – Counting and Cardinality**

| **New Math Standards** | **Grade Level Expectations** | **Comment** |
| --- | --- | --- |
| **Counting and Cardinality K.CC** |  |  |
| **Know number names and the count sequence.** |  |  |
| K.CC.1. Count to 100 by ones and by tens. | **The student demonstrates conceptual understanding**  • **of whole numbers to 20 by**  **[K] N-2** recognizing and counting whole numbers from 0-20  **[K] N-12** demonstrating skip counting by 2’s, 5’s, and 10’s with support | GLEs count only to 20 in kindergarten but grade 1 GLEs have students count to 100.  **The student demonstrates conceptual understanding**   * **of whole numbers to one hundred by**   **[1] N-1** reading, writing, ordering/counting and modeling correspondence of whole numbers  **The student demonstrates conceptual understanding of number theory by**  **[1] N-8** skip counting by 2’s to 20 and 5’s and 10’s to 100  GLE also skip counts by 2’s and 5’s. |
| K.CC.2. Count forward beginning from a given number within the known sequence. | NEW – NOT ADDRESSED IN THE GLES |  |
| K.CC.3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). | **The student demonstrates conceptual understanding**  • **of whole numbers to 20 by**  **[K] N-1** demonstrating 1-1 correspondence  **[K] N-3** writing and ordering whole numbers from 0-20 |  |
| **Count to tell the number of objects**. |  |  |
| K.CC.4. Understand the relationship between numbers and quantities to cardinality.  a. When counting objects, say the number names in standard order, pairing each object with one and only one number name and each number name with one and only one object.  b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.  c. Understand that each successive number name refers to a quantity that is one larger. | a. **[K] N-1** demonstrating 1-1 correspondence  b. NEW – NOT ADDRESSED IN THE GLES  c. NEW – NOT ADDRESSED IN THE GLES | b. The new standard specifies “regardless of their arrangement or order in which counted.” |
| K.CC.5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects. | **[K] N-1** demonstrating 1-1 correspondence | The new standard gives more specific details. The GLE is global and can encompass many skills. |
| **Compare numbers.** |  |  |
| K.CC.6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group (e.g., by using matching, counting, or estimating strategies). | **[K] F&R-5** showing more, less, or equal to using objects |  |
| K.CC.7. Compare and order two numbers between 1 and 10 presented as written numerals. | **The student demonstrates conceptual understanding**  • **of whole numbers to 20 by**  **[K] N-3** writing and ordering whole numbers from 0-20 | The new standard includes the skill of comparing numbers, which could be considered as part of ordering in the GLE. |

**Alaska New Mathematics Standards – Operations and Algebraic Thinking**

| **New Math Standards** | **Grade Level Expectations** | **Comment** |
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| **Operations and Algebraic Thinking K.OA** |  |  |
| **Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.** |  |  |
| K.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps) acting out situations, verbal explanations, expressions, or equations. | **The student demonstrates conceptual understanding of mathematical operations by**  **[K] N-10** using objects or pictures to model addition and subtraction |  |
| K.OA.2. Add or subtract whole numbers to 10 (e.g., by using objects or drawings to solve word problems). | **The student demonstrates algebraic thinking by**  **[K] F&R-4** adding or subtracting whole numbers to 10 using manipulatives to solve story problems |  |
| K.OA.3. Decompose numbers less than or equal to 10 into pairs in more than one way (e.g., by using objects or drawings, and record each decomposition by a drawing or equation). | NEW – NOT ADDRESSED IN THE GLES | Decomposing numbers is not addressed in GLES – the closest GLEs dealing with adding and subtracting are below.  **[K] E&C-3** adding and subtracting whole numbers up to ten using manipulatives  **[K] F&R-4** adding or subtracting whole numbers to 10 using manipulatives to solve story problems |
| K.OA.4. For any number from 1-~~4~~, find the number that makes 5 when added to the given number and, for any number from 1-9, find the number that makes 10 when added to the given number (e.g., by using objects, drawings or 10 frames) and record the answer with a drawing or equation. | NEW – NOT ADDRESSED IN THE GLES | GLEs do not specifically address addends of 10 or missing addends. At grade 2, the GLE addresses missing addends.  **The student demonstrates algebraic thinking by**  **[2] F&R-3** solving a problem with an unknown (e.g., 7 + ? = 10) |
| K.OA.5. Fluently add and subtract numbers up to 5. | NEW – NOT ADDRESSED IN THE GLES | Adding and subtracting is in the grade 1 GLEs.  **The student accurately solves problems (including real-world situations) involving**  **[1] E&C-3** recalling addition and subtraction facts 0-10 |
| **Identify and continue patterns.** |  |  |
| K.OA.6. Recognize, identify and continue simple patterns of color, shape, and size. | **[K] F&R-3** recognizing, identifying, and continuing simple patterns of color, shape, or size |  |

**Alaska New Mathematics Standards – Number and Operations in Base Ten**

| **New Math Standards** | **Grade Level Expectations** | **Comment** |
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| **Number and Operations in Base Ten K.NBT** |  |  |
| **Work with numbers 11-19 to gain foundations for place value.** |  |  |
| K.NBT.1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones (e.g., by using objects or drawings) and record each composition and decomposition by a drawing or equation (e.g., 18=10 +8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight or nine ones. | NEW – NOT ADDRESSED IN THE GLES | The new standard has a focus on decomposing numbers in the teens. The GLEs do not have an equivalent focus on the Number and Operations in Base Ten. |

**Alaska New Mathematics Standards – Measurement and Data**

| **New Math Standards** | **Grade Level Expectations** | **Comment** |
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| **Measurement and Data K.MD** |  |  |
| **Describe and compare measurable attributes.** |  |  |
| K.MD.1. Describe measurable attributes of objects (e.g., length or weight). Match measuring tools to attribute (e.g., ruler to length). Describe several measureable attributes of a single object. | **[K] MEA-3** identifying instruments used to measure length, time, and temperature | The GLE does not ask students to describe measureable attributes and does not specify weight. |
| K.MD.2. Make comparisons between two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* | **The student demonstrates understanding of measurable attributes by**  **[K] MEA-1** making comparisons between objects using concepts of big/little, long/short, large/small, more/less, same |  |
| **Classify objects and count the number of objects in each category.** |  |  |
| K.MD.3. Classify objects into given categories (attributes). Count the number of objects in each category (limit category counts to be less than or equal to 10). | **The student demonstrates conceptual understanding of functions, patterns, or sequences by**  **[K] F&R-2** identifying, sorting, and classifying objects by attribute and identifying objects that do not belong to a particular group  **The student determines reasonable answers to real-life situations, paper/pencil computations, or calculator results by**  **[K] E&C-1** comparing the number of objects in different sets using more, less, same |  |
| **Work with time and money.** |  |  |
| K.MD.4. Name in sequence the days of the week. | **[K] MEA-4** naming in sequence the days of the week |  |
| K.MD.5 Tell time to the hour using both analog and digital clocks. | **[K] MEA-5** telling time to the hour using analog and digital clocks |  |
| K.MD.6. Identify coins by name. | **[K] MEA-2** identifying coins by name: penny, nickel, dime, and quarter |  |

**Alaska New Mathematics Standards – Geometry**

| **New Math Standards** | **Grade Level Expectations** | **Comment** |
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| **Geometry K.G** |  |  |
| **Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).** |  |  |
| K.G.1. Describe objects in the environment using names of shapes and describe their relative positions (e.g., *above*, *below*, *beside*, *in front of*, *behind*, *next to)*. | **The student demonstrates understanding of position and direction by**  **[K] G-5** identifying positions of objects that are above, below, before, after, next to, in the middle of, in front of, behind…  **[K] G-3** identifying triangle, circle, rectangle, and square | The GLE is limited to four shapes but includes more positions and directions. |
| K.G.2. Name shapes regardless of their orientation or overall size. | **[K] G-3** identifying triangle, circle, rectangle, and square | The GLE does not refer to orientation or overall size. |
| K.G.3. Identify shapes as two-dimensional or three-dimensional. | NEW – NOT ADDRESSED IN THE GLES | The GLEs do not address three-dimensional shapes until grade 2.  **The student demonstrates an understanding of geometric relationships by**  **[2] G-2** identifying and classifying 3-dimensional shapes (e.g., cone, sphere and cylinder) |
| **Analyze, compare, create, and compose shapes.** |  |  |
| K.G.4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices), and other attributes (e.g., having sides of equal lengths). | **The student demonstrates an understanding of geometric relationships by**  **[K] G-1** sorting and classifying shapes according to similar attributes  **The student demonstrates conceptual understanding of similarity, congruence, symmetry, or transformations of shapes by**  **[K] G-4** comparing geometric shapes | The new standard is addressed more closely in the grades 1 and 2 GLEs.  **[1] G-1** identifying the attributes of 2-dimensional shapes (e.g., a triangle has three sides)  **[1] G-2** identifying and classifying 2 dimensional shapes through visual observations and properties (e.g., which of these shapes is a triangle)  **[2] G-2** identifying and classifying 3-dimensional shapes (e.g., cone, sphere and cylinder) |
| K.G.5. Build shapes (e.g., using sticks and clay) and draw shapes. | **The student demonstrates a conceptual understanding of geometric drawings or constructions by**  **[K] G-6** drawing, copying, or describing triangles, squares, rectangles and circles | The new standard includes the building of shapes, which is addressed in the grade 2 GLE.  **[2] G-5** creating simple shapes using concrete materials/manipulatives) |
| K.G.6. Put together two-dimensional shapes to form larger shapes (e.g., join two triangles with full sides touching to make a rectangle). | NEW – NOT ADDRESSED IN THE GLES |  |

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| **Kindergarten Math GLEs Not Matched by New Standards** | **Comments** |
| **The student demonstrates conceptual understanding of whole numbers to 20 by**  **[K] N-4** counting whole numbers backwards from 10 to 0 | New Kindergarten standard (**K.OA.2)** involving subtraction is related and could be linked to instruction for the standard. |
| **[K] N-5** identifying ordinal position, first to the tenth | New 1st Grade standard (**1.CC.2).** |
| **The student demonstrates conceptual understanding of simple fractions**  **[K] N-6** dividing an even numbered set of concrete objects (up to 20) into halves | Fractions are addressed in the new 3rd Grade standards. |
| **[K] N-7** identifying halves |
| **[K] N-8** identifying full, half full, and empty containers |
| **The student demonstrates conceptual understanding of mathematical operations by**  **[K] N-9** recognizing (+), (–), and (=) signs | New 1st Grade standard **(1.OA.7)** including understanding the equal sign is related. |
| **[K] N-11** using number lines or objects related to real situations | New 1st Grade standard **(1.OA.1)** using a number line is related. |
| **The student determines reasonable answers to real-life situations, paper/pencil computations, or calculator results by**  **[K] E&C-2** estimating the number of objects in a given set as more or less than 10 | New 1st Grade standard **(1.CC.6)** includes estimating to 20 is related. |
| **The student accurately solves problems (including real-world situations) involving**  **[K] E&C-3** adding and subtracting whole numbers up to ten using manipulatives | New Kindergarten standard **(K.OA.3)** about decomposing numbers is related. |
| **The student demonstrates conceptual understanding of functions, patterns, or sequences by**  **[K] F&R-1** recognizing patterns found in common objects, sounds, and movements | New Kindergarten standard **(K.OA.6)** partially addresses simple patterns of color, shape and size. |
| **The student demonstrates an understanding of geometric relationships by**  **[K] G-2** describing objects using three attributes such as size, color, and shape | New 1st Grade standard **(1.G.1)** address defining and non-defining attributes is related. |
| **The student demonstrates an ability to classify and organize data by**  **[K] S&P-1** constructing real graphs using concrete objects or pictographs with support | New 1st Grade standard **(1.MD.7)** addressing data representation is related. |
| **[K] S&P-2** collecting and recording data with support |
| **The student demonstrates an ability to analyze data (comparing, explaining, interpreting, evaluating; or drawing or justifying conclusions) by**  **[K] S&P-3** describing information from real graphs or pictographs |
| **The student demonstrates a conceptual understanding of probability and counting techniques by**  **[K] S&P-4** making simple predictions using events or repeated observations | Probability is addressed in by the new Grade 6 standards. |
| **The student demonstrates an ability to problem solve by**  **[K] PS-1** solving simple problems using concrete objects | The GLE math process skills are incorporated in to the Standards for Mathematical Practice.   1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.     Descriptions of the Standards for Mathematical Practice follow this chart as well as the grade-span descriptors appropriate to this grade level. |
| **The student communicates his or her mathematical thinking by**  **[K] PS-2** telling how objects were used to solve simple problems |
| **The student demonstrates an ability to use logic and reason by**  **[K] PS-3** explaining what makes sense |
| **[K] PS-4** drawing pictures that support simple mathematical statements |
| **The student understands and applies mathematical skills and processes across the content strands by**  **[K] PS-5** using real world context (i.e., self, friends, and family) |

**Alaska New Standards for Mathematical Practice**

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| **1. Make sense of problems and persevere in solving them.**  Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. | **In grades K-2 mathematically proficient students will:**   1. focus on the problem and check for alternate methods 2. check if the solution makes sense |

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| **2. Reason abstractly and quantitatively.**  Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. | **In grades K-2 mathematically proficient students will:**   1. represent a situation symbolically and/or with manipulatives 2. create a coherent representation of the problem 3. use units of measurement consistently |
| **3. Construct viable arguments and critique the reasoning of others.**  Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. | **In grades K-2 mathematically proficient students will:**   1. construct arguments using concrete referents such as objects, drawings, diagrams, and actions 2. justify conclusions, communicate conclusions 3. listen to arguments and decide whether the arguments make sense |

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| **4. Model with mathematics.**  Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two‐way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. | **In grades K-2 mathematically proficient students will:**   1. apply mathematics to solve problems in everyday life 2. identify important quantities in a practical situation and model the situation with manipulatives or pictures 3. interpret mathematical results in the context of the situation and reflect on whether the results make sense |

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| **5. Use appropriate tools strategically.**  Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts. | **In grades K-2 mathematically proficient students will:**   1. select the available tools (such as pencil and paper, manipulatives, rulers, and available technology) when solving a mathematical problem 2. be familiar with tools appropriate for the grade level to make sound decisions about when each of these tools might be helpful 3. identify relevant external mathematical resources and use them to pose or solve problems 4. use technological tools to explore and deepen their understanding of concepts |

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| **6. Attend to precision.**  Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. | **In grades K-2 mathematically proficient students will:**   1. give thoughtful explanations to each other 2. use clear definitions and reasoning in discussion with others 3. state the meaning of symbols they choose, including using the equal sign consistently and appropriately |

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| **7. Look for and make use of structure.**  Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression *x*2 + 9*x* + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3*(x* – *y*)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*. | **In all grade levels mathematically proficient students will:**   * discern a pattern or structure * understand complex structures as single objects or as being composed of several objects * check if the answer is reasonable |

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| **8. Look for and express regularity in repeated reasoning.**    Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (*y* – 2)/(*x* – 1) = 3. Noticing the regularity in the way terms cancel when expanding (*x* – 1)(*x* + 1), (*x* – 1)(*x*2 + *x* + 1), and (*x* – 1)(*x*3 + *x*2 + *x* + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. | **In all grade levels mathematically proficient students will:**   * identify if calculations or processes are repeated * use alternative and traditional methods to solve problems * evaluate the reasonableness of their intermediate results, while attending to the details |