Department of Education & Early Development



# ALASKA ENGLISH/LANGUAGE ARTS AND MATHEMATICS STANDARDS

Adopted June 2012



# Alaska Board of Education & Early Development

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# Alaska English/Language Arts and Mathematics Content Standards

High academic standards are an important first step in ensuring that all Alaska's students have the tools they need for success. These standards reflect the collaborative work of Alaskan educators and national experts from the nonprofit National Center for the Improvement of Educational Assessment. Further, they are informed by public comments. Alaskan teachers have played a key role in this effort, ensuring that the standards reflect the realities of the classroom. Since work began in spring 2010, the standards have undergone a thoughtful and rigorous drafting and refining process.

A nationwide movement among the states and employers has called for America's schools to prepare students to be ready for postsecondary education and careers. Standards in English/language arts and mathematics build a foundation for college and career readiness. Students proficient in the standards read widely and deeply in a range of subjects, communicate clearly in written and spoken English, have the capacity to build knowledge on a subject, and understand and use mathematics.

Industry leaders were part of Alaska's standards review. Repeatedly these leaders placed the greatest weight on critical thinking and adaptability as essential skills in the workplace. Industry leaders believe that strengthening our K-12 system will help ensure that Alaskans are prepared for high-demand, good-wage jobs. Instructional expectations that include employability standards will help students prepare for a career.

Additionally, institutions of higher education were engaged in refining Alaska's standards. These educators focused on whether the standards would culminate in student preparedness. Students proficient in Alaska's standards will be prepared for credit-bearing courses in their first year of postsecondary education. It is critical that students can enter institutions of higher education ready to apply their knowledge, extend their learning, and gain technical and jobrelated skills.

These standards do not tell teachers how to teach, nor do they attempt to override the unique qualities of each student and classroom. They simply establish a strong foundation of knowledge and skills all students need for success after graduation. It is up to schools and teachers to decide how to put the standards into practice and incorporate other state and local standards, including cultural standards. In sum, students must be provided opportunities to gain skills and learn to apply them to real-world life and work situations.

# **Introduction to English/Language Arts Standards**

#### Reading

The standards establish increasing complexity in what students must be able to read so all students are ready for the demands of college-level and career-level reading no later than the end of high school. The standards also require the progressive development of reading comprehension; students advancing through the grades are able to gain more from whatever they read.

Through reading a diverse array of classic, contemporary, and Alaskan-based literature as well as challenging informational texts, students are expected to build knowledge, gain insights, explore possibilities, and broaden their perspective. This may require a review of texts provided at various grades, and within courses, to determine if the full breadth of reading is available.

The reading standards in K-5 include Foundational Skills. The Foundational Skills are focused on developing students' understanding and working-knowledge of print concepts, phonological awareness, phonics and word recognition, and fluency. A key design feature is that at the same time students are developing strong Foundational Skills (learning to read well) they are also developing strong comprehension and vocabulary skills by listening to and reading stories and informational texts about animals, space, or the history of where they live.

The reading standards place equal emphasis on the sophistication of what students read and the skill with which they read. Standard 10 defines a grade-by-grade "staircase" of increasing text complexity that rises from beginning reading to the college- and career-readiness level. Teachers are to engage students in a range of text at multiple grade levels; an extension into upper grade levels may require scaffolding. Whatever they are reading, students must also show a steadily growing ability to discern more from and make fuller use of the text, including making an increased number of connections among ideas and between texts, considering a wider range of textual evidence, and becoming more sensitive to inconsistencies, ambiguities, and poor reasoning in texts.

#### Writing

The ability to write logical arguments based on substantive claims, sound reasoning, and relevant evidence is a cornerstone of the writing standards, with opinion writing—a basic form of argument—extending down into the earliest grades.

Student research includes both short, focused projects and longer-term, in-depth projects. This is emphasized throughout the standards. Research skills are predominantly in the writing strand since a written analysis and presentation of findings are so often critical to communicate information.

#### Speaking and Listening

The standards require that students gain, evaluate, and present increasingly complex information, ideas, and evidence through listening and speaking as well as through media.

An important focus of the speaking and listening standards is academic discussion in one-on-one, small-group, and whole-class settings. Formal presentations are important, but so is the more informal discussion that takes place as students collaborate to answer questions, build understanding, and solve problems.

#### Language

The standards provide opportunities for students to develop their vocabularies through a mix of conversations, direct instruction, and reading. The standards emphasize word meanings and nuances of words, and steadily expand the repertoire of words and phrases.

The language standards prepare students for real life experience at college and in twenty-first century careers. Students must be able to use formal English in their writing and speaking and be able to make informed, skillful choices among the many ways to express themselves through language.

A Language Progressive Skills table accompanies the language standards. The table shows language standards introduced in each grade that are particularly likely to require continued attention in higher grades as they are applied to increasingly sophisticated writing and speaking. In other words, even though the standards noted in the Language Progressive Skills table are not repeated in higher grades, they must be incorporated into instruction.

#### Literacy development across the curriculum

The literacy standards establish that interaction in reading, writing, speaking, listening, and language be shared responsibly within the school. The K-5 standards include expectations for reading, writing, speaking, listening, and language applicable to a range of subjects, including but not limited to English/language arts. The grades 6-12 standards are divided into two sections, one for English/language arts and the other for history/social studies, science, and technical subjects. This division recognizes that teachers in other content areas must have a role in the development of students' literacy skills.

# Students Who are College and Career Ready in Reading, Writing, Speaking, Listening, and Language

The descriptions that follow are not standards themselves but instead offer a portrait of students who meet the standards set out in this document. As students advance through the grades and master the standards in reading, writing, speaking, listening, and language, they are able to exhibit with increasing fullness and regularity these capacities of the literate individual.

#### They demonstrate independence.

Students can, without significant scaffolding, comprehend and evaluate complex texts across a range of types and disciplines, and they can construct effective arguments and convey intricate or multifaceted information. Likewise, students are able independently to discern a speaker's key points, request clarification, and ask relevant questions. They build on others' ideas, articulate their own ideas, and confirm they have been understood. Without prompting, they demonstrate command of Standard English and acquire and use a wide-ranging vocabulary. More broadly, they become self-directed learners, effectively seeking out and using resources to assist them, including teachers, peers, and print and digital reference materials.

#### They build strong content knowledge.

Students establish a base of knowledge across a wide range of subject matter by engaging with works of quality and substance. They become proficient in new areas through research and study. They read purposefully and listen attentively to gain both general knowledge and discipline-specific expertise. They refine and share their knowledge through writing and speaking.

#### They respond to the varying demands of audience, task, purpose, and discipline.

Students adapt their communication in relation to audience, task, purpose, and discipline. They set and adjust purpose for reading, writing, speaking, listening, and language use as warranted by the task. They appreciate nuances, such as how the composition of an audience should affect tone when speaking and how the connotations of words affect meaning. They also know that different disciplines call for different types of evidence (e.g., documentary evidence in history, experimental evidence in science).

#### They comprehend as well as critique.

Students are engaged and open-minded—but discerning—readers and listeners. They work diligently to understand precisely what an author or speaker is saying, but they also question an author's or speaker's assumptions and premises and assess the veracity of claims and the soundness of reasoning.

#### They value evidence.

Students cite specific evidence when offering an oral or written interpretation of a text. They use relevant evidence when supporting their own points in writing and speaking, making their reasoning clear to the reader or listener, and they constructively evaluate others' use of evidence.

## They use technology and digital media strategically and capably.

Students employ technology thoughtfully to enhance their reading, writing, speaking, listening, and language use. They tailor their searches online to acquire useful information efficiently, and they integrate what they learn using technology with what they learn offline. They are familiar with the strengths and limitations of various technological tools and media and can select and use those best suited to their communication goals.

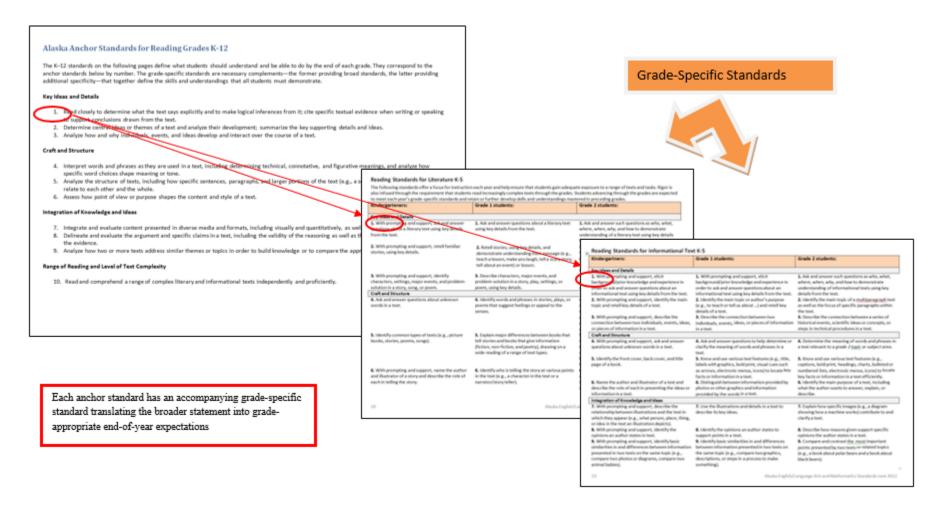
#### They come to understand other perspectives and cultures.

Students appreciate that the twenty-first-century classroom and workplace are settings in which people from often widely divergent cultures and who represent diverse experiences and perspectives must learn and work together. Students actively seek to understand other perspectives and cultures through reading and listening, and they are able to communicate effectively with people of varied backgrounds. They evaluate other points of view critically and constructively. Through reading great classic and contemporary works of literature representative of a variety of periods, cultures, and worldviews, students can vicariously inhabit worlds and have experiences much different than their own.

# Organization of English/Language Arts Standards

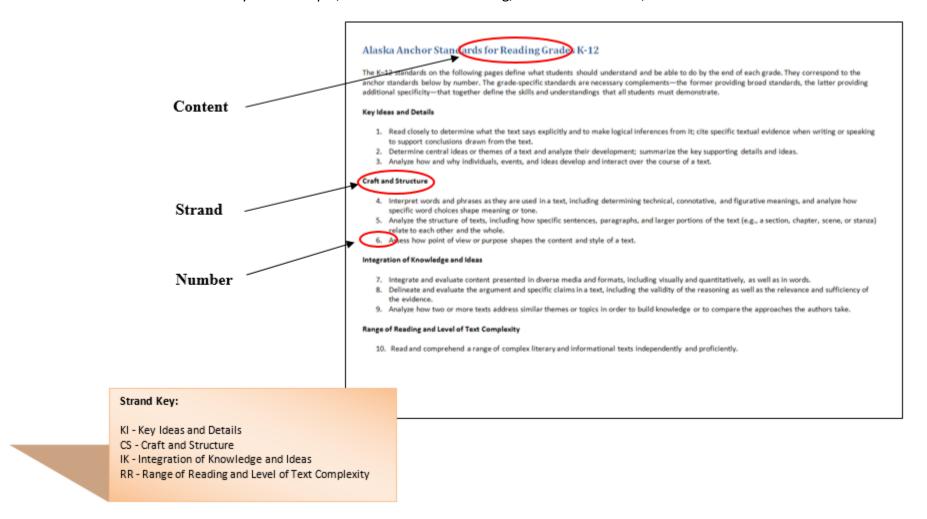
The Standards comprise two main sections: a comprehensive K-5 section and content area-specific section for grades 6-12. Appendices and instructional tools accompany the main document and can be found on the state's website http://www.eed.alaska.gov.

Standards for each grade within K-8 and for grade spans 9-10 and 11-12 follow the same anchor standards for each content area: reading, writing, listening and speaking, and language. Each grade-specific standard corresponds to the same-numbered anchor standard. Put another way, each anchor standard has an accompanying grade-specific standard translating the broader statement into grade-appropriate end-of-year expectations. Anchor standards "anchor" the document and define general, cross-disciplinary literacy expectations.



The K-12 grade-specific standards define end-of-year expectations and a cumulative progression designed to enable students to meet college and career readiness. Individual grade-specific standards can be identified by their content/focus, grade, strand, and number (or number and letter, where applicable), so that RI.4.3, for example, stands for Reading, Informational Text, grade 4, standard 3, and W.5.1a stands for Writing, grade 5, standard 1a.

Anchor standards are coded similarly. For example, R.CS.6 stands for Reading, Craft and Structure, standard 6.



# Alaska English/Language Arts Anchor Standards

# **Alaska Anchor Standards Reading Grades K-12**

The K-12 grade-specific standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the anchor standards below by number. The grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

#### **Key Ideas and Details**

- 1. Read closely to determine what the text says explicitly and to make logical inferences from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
- 2. Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
- 3. Analyze how and why individuals, events, and ideas develop and interact over the course of a text.

#### **Craft and Structure**

- 4. Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
- 5. Analyze the structure of texts, including how specific sentences, paragraphs, and larger portions of the text (e.g., a section, chapter, scene, or stanza) relate to each other and the whole.
- 6. Assess how point of view or purpose shapes the content and style of a text.

## Integration of Knowledge and Ideas

- 7. Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
- 8. Delineate and evaluate the argument and specific claims in a text, including the validity of the reasoning as well as the relevance and sufficiency of the evidence.
- 9. Analyze how two or more texts address similar themes or topics in order to build knowledge or to compare the approaches the authors take.

## Range of Reading and Level of Text Complexity

10. Read and comprehend a range of complex literary and informational texts independently and proficiently.

# Measuring Text Complexity: Three Factor

- 1. Quantitative measures readability and other scores of text complexity, often best measured by computer software.
- 2. Qualitative measures levels of meaning, structure, language conventionality and clarity, and knowledge demands, often best measured by an attentive human reader.
- 3. Reader and Task considerations background knowledge of reader, motivation, interests, and complexity generated by tasks assigned, often best made by educators employing their professional judgment.



Text complexity described in Standard 10 is not limited to the quantitative measure, which deals exclusively with length and number of words. The more rigorous standard considers a qualitative measure, which addresses the levels of meaning, structure, language conventionality and clarity. Reader and task considerations look at the background knowledge of reader, motivation and interest. For instance, Steinbeck's *Grapes of Wrath* may be considered at the second-grade level when looking only at the quantitative measure, but when the language conventions and background knowledge of the reader are considered, it is a ninth grade level.

## Range of Text Types for K-5

Students in K-5 apply the Reading standards to the following range of text types, with texts selected from a broad range of cultures and periods.

Literature Informational Text

Stories	Dramas	Poetry	Literary Nonfiction and Historical, Scientific, Technical Texts
Includes children's	Includes staged	Includes nursery rhymes	Includes biographies and autobiographies; books about history,
adventure stories, folktales,	dialogue and brief	and the subgenres of the	social studies, science, and the arts; technical texts, including
legends, fables, fantasy,	familiar scenes	narrative poem,	directions, forms, and information displayed in graphs, charts,
realistic fiction, and myth		limerick, and free verse	or maps; and digital sources on a range of topics
		poem	

## Range of Text Types for 6-12

Students in 6-12 apply the Reading standards to the following range of text types, with texts selected from a broad range of cultures and periods.

## Literature Informational Text

Stories	Dramas	Poetry	Literary Nonfiction and Historical, Scientific, Technical Texts
Includes the subgenres of	Includes one-act	Includes the subgenres	Includes the subgenres of exposition, argument, and functional
adventure stories, historical	and multi-act	of narrative poems,	text in the form of personal essays, speeches, opinion pieces,
fiction, mysteries, myths,	plays, both in	lyrical poems, free verse	essays about art or literature, biographies, memoirs, journalism,
science fiction, realistic	written form and	poems, sonnets, odes,	and historical, scientific, technical, or economic accounts
fiction, allegories, parodies,	on film	ballads, and epics	(including digital sources) written for a broad audience
satire, and graphic novels			

# Alaska Anchor Standards for Writing Grades K-12

The K-12 grade-specific standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the anchor standards below by number. The grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

#### **Text Types and Purposes**

- 1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
- 2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
- 3. Use narrative writing to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences.

#### **Production and Distribution of Writing**

- 4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
- 5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.
- 6. Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.

#### **Research to Build and Present Knowledge**

- 7. Conduct short as well as more sustained research projects based on focused questions, demonstrating understanding of the subject under investigation.
- 8. Gather relevant information from multiple print and digital sources, assess the credibility and accuracy of each source, and integrate the information while avoiding plagiarism.
- 9. Draw evidence from literary or informational texts to support analysis, reflection, and research.

## Range of Writing

10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.

## Alaska Anchor Standards for Speaking and Listening Grades K-12

The K-12 grade-specific standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the anchor standards below by number. The grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

## **Comprehension and Collaboration**

- 1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
- 2. Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.
- 3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric.

#### **Presentation of Knowledge and Ideas**

- 4. Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and that the organization, development, and style are appropriate to task, purpose, and audience.
- 5. Make strategic use of digital media and visual displays of data to express information and enhance understanding of presentations.
- 6. Adapt speech to a variety of contexts and communicative tasks, demonstrating command of formal English when indicated or appropriate.

# Alaska Anchor Standards for Language Grades K-12

The K-12 grade-specific standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the anchor standards below by number. The grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

#### **Conventions of Standard English**

- 1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.
- 2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.

## **Knowledge of Language**

3. Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening.

#### **Vocabulary Acquisition and Use**

- 4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
- 5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings.
- 6. Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking, and listening at the college- and career-readiness level; demonstrate independence in gathering vocabulary knowledge when encountering an unknown term important to comprehension or expression.

# **Language Progressive Skills, by Grade**

The following skills, marked with an asterisk (\*) in Language standards 1–3, are particularly likely to require continued attention in higher grades as they are applied to increasingly sophisticated writing and speaking.

Standard								
Grade Level:	3	4	5	6	7	8	9-10	11-12
L.3.1f. Ensure subject-verb and pronoun-antecedent agreement.								
L.3.3a. Choose words and phrases for effect.								
<b>L.4.1f.</b> Produce complete sentences, recognizing and correcting inappropriate fragments								
and run-ons.								
<b>L.4.1g.</b> Correctly use frequently confused words (e.g., to/too/two; there/their).								
L.4.3a. Choose words and phrases to convey ideas precisely.*								
<b>L.4.3b.</b> Choose punctuation for effect.								
<b>L.5.1d.</b> Recognize and correct inappropriate shifts in verb tense.								
L.5.2a. Use punctuation to separate items in a series.†								
<b>L.6.1c.</b> Recognize and correct inappropriate shifts in pronoun number and person.								
<b>L.6.1d.</b> Recognize and correct vague pronouns (i.e., ones with unclear or ambiguous								
antecedents).								
<b>L.6.1e.</b> Recognize variations from standard English in their own and others' writing and								
speaking, and identify and use strategies to improve expression in conventional language.								
<b>L.6.2a.</b> Use punctuation (commas, parentheses, dashes) to set off								
nonrestrictive/parenthetical elements.								
<b>L.6.3a.</b> Vary sentence patterns for meaning, reader/listener interest, and style.‡								
L.6.3b. Maintain consistency in style and tone.								
<b>L.7.1c.</b> Place phrases and clauses within a sentence, recognizing and correcting misplaced								
and dangling modifiers.								
<b>L.7.3a.</b> Choose language that expresses ideas precisely and concisely, recognizing and								
eliminating wordiness and redundancy.								
<b>L.8.1d.</b> Recognize and correct inappropriate shifts in verb voice and mood.								
L.9–10.1a. Use parallel structure.								

<sup>\*</sup>Subsumed by L.7.3a, †Subsumed by L.9–10.1a, ‡Subsumed by L.11–12.3a

# **K-5 English/Language Arts Standards**

# **Reading Standards for Literature K-5**

The following standards offer a focus for instruction each year and help ensure that students gain adequate exposure to a range of texts and tasks. Rigor is also infused through the requirement that students read increasingly complex texts through the grades. Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.

Kindergartners:	Grade 1 students:	Grade 2 students:
Key Ideas and Details	Key Ideas and Details	Key Ideas and Details
1. With prompting and support, ask and	1. Ask and answer questions about a literary	1. Ask and answer such questions as who,
answer questions about a literary text using	text using key details from the text.	what, where, when, why, and how to
key details from the text.		demonstrate understanding of a literary text
		using key details from the text.
2. With prompting and support, retell familiar	2. Retell stories, using key details, and	2. Retell stories, including fables and folktales
stories, using key details.	demonstrate understanding their message	from diverse cultures, and determine the
	(e.g., teach a lesson, make you laugh, tell a	author's purpose (e.g., teach a lesson, make
	scary story, tell about an event) or lesson.	you laugh, tell a scary story, describe an
		imaginary place), lesson or moral.
3. With prompting and support, identify	<b>3.</b> Describe characters, settings, major events,	<b>3.</b> Describe how characters in a story, play or
characters, settings, major events, and	and problem-solution in a story, play, or	poem respond to major events, problems,
problem-solution in a story, song, or poem.	poem, using key details.	and challenges.
Craft and Structure	Craft and Structure	Craft and Structure
Craft and Structure 4. Ask and answer questions about unknown	Craft and Structure 4. Identify words and phrases in stories,	Craft and Structure 4. Identify words and phrases that supply
4. Ask and answer questions about unknown	4. Identify words and phrases in stories,	4. Identify words and phrases that supply
4. Ask and answer questions about unknown	<b>4.</b> Identify words and phrases in stories, plays, or poems that suggest feelings or	<b>4.</b> Identify words and phrases that supply rhythm or sensory images and meaning in a
4. Ask and answer questions about unknown	<b>4.</b> Identify words and phrases in stories, plays, or poems that suggest feelings or	<b>4.</b> Identify words and phrases that supply rhythm or sensory images and meaning in a story, poem, or song (e.g., regular beats,
4. Ask and answer questions about unknown	<b>4.</b> Identify words and phrases in stories, plays, or poems that suggest feelings or appeal to the senses.	<b>4.</b> Identify words and phrases that supply rhythm or sensory images and meaning in a story, poem, or song (e.g., regular beats, alliteration, rhymes, repeated lines) and
4. Ask and answer questions about unknown	<b>4.</b> Identify words and phrases in stories, plays, or poems that suggest feelings or	<b>4.</b> Identify words and phrases that supply rhythm or sensory images and meaning in a story, poem, or song (e.g., regular beats, alliteration, rhymes, repeated lines) and describe how they make a reader feel or what
<b>4.</b> Ask and answer questions about unknown words in a text.	<b>4.</b> Identify words and phrases in stories, plays, or poems that suggest feelings or appeal to the senses.	4. Identify words and phrases that supply rhythm or sensory images and meaning in a story, poem, or song (e.g., regular beats, alliteration, rhymes, repeated lines) and describe how they make a reader feel or what a reader might see in his or her mind.
<ul><li>4. Ask and answer questions about unknown words in a text.</li><li>5. Identify common types of texts (e.g.,</li></ul>	<ul><li>4. Identify words and phrases in stories, plays, or poems that suggest feelings or appeal to the senses.</li><li>5. Explain major differences between books</li></ul>	4. Identify words and phrases that supply rhythm or sensory images and meaning in a story, poem, or song (e.g., regular beats, alliteration, rhymes, repeated lines) and describe how they make a reader feel or what a reader might see in his or her mind.  5. Describe the overall structure of a story,
<ul><li>4. Ask and answer questions about unknown words in a text.</li><li>5. Identify common types of texts (e.g.,</li></ul>	<ul> <li>4. Identify words and phrases in stories, plays, or poems that suggest feelings or appeal to the senses.</li> <li>5. Explain major differences between books that tell stories and books that give</li> </ul>	<ul> <li>4. Identify words and phrases that supply rhythm or sensory images and meaning in a story, poem, or song (e.g., regular beats, alliteration, rhymes, repeated lines) and describe how they make a reader feel or what a reader might see in his or her mind.</li> <li>5. Describe the overall structure of a story, including describing how the beginning introduces the story (who, what, why, where), the middle describes the problem</li> </ul>
<ul><li>4. Ask and answer questions about unknown words in a text.</li><li>5. Identify common types of texts (e.g.,</li></ul>	<ul> <li>4. Identify words and phrases in stories, plays, or poems that suggest feelings or appeal to the senses.</li> <li>5. Explain major differences between books that tell stories and books that give information (fiction, non-fiction, and poetry),</li> </ul>	<ul> <li>4. Identify words and phrases that supply rhythm or sensory images and meaning in a story, poem, or song (e.g., regular beats, alliteration, rhymes, repeated lines) and describe how they make a reader feel or what a reader might see in his or her mind.</li> <li>5. Describe the overall structure of a story, including describing how the beginning introduces the story (who, what, why,</li> </ul>
<ul><li>4. Ask and answer questions about unknown words in a text.</li><li>5. Identify common types of texts (e.g.,</li></ul>	<ul> <li>4. Identify words and phrases in stories, plays, or poems that suggest feelings or appeal to the senses.</li> <li>5. Explain major differences between books that tell stories and books that give information (fiction, non-fiction, and poetry), drawing on a wide reading of a range of text</li> </ul>	<ul> <li>4. Identify words and phrases that supply rhythm or sensory images and meaning in a story, poem, or song (e.g., regular beats, alliteration, rhymes, repeated lines) and describe how they make a reader feel or what a reader might see in his or her mind.</li> <li>5. Describe the overall structure of a story, including describing how the beginning introduces the story (who, what, why, where), the middle describes the problem</li> </ul>

Kindergartners:	Grade 1 students:	Grade 2 students:
6. With prompting and support, name the	<b>6.</b> Identify who is telling the story at various	<b>6.</b> Acknowledge differences in the points of
author and illustrator of a story and describe	points in the text (e.g., a character in the text	view of characters, including by speaking in a
the role of each in telling the story.	or a narrator/story teller).	different voice for each character when
		reading dialogue aloud.
Integration of Knowledge and Ideas	Integration of Knowledge and Ideas	Integration of Knowledge and Ideas
7. With prompting and support, describe the	7. Use illustrations and details in a story read	7. Use information gained from the
relationship between illustrations and the	or read aloud to describe its characters,	illustrations and words in a print or digital text
story in which they appear (e.g., what	events, setting, or problem-solution.	to demonstrate understanding of its
moment in a story an illustration depicts) or		characters, setting, or plot (e.g., problem-
use illustrations to tell or retell a story.		solution; chronology).
8. (Not applicable to literature)	8. (Not applicable to literature)	8. (Not applicable to literature)
<b>9.</b> With prompting and support, compare and	<b>9.</b> With prompting and support, compare and	9. Compare and contrast two or more
contrast the adventures and experiences of	contrast the adventures and experiences of	versions of the same story/text (e.g.,
characters in familiar stories.	characters in stories.	Cinderella stories) by different authors or
		from different cultures.
Range of Reading and Level of Complexity	Range of Reading and Level of Complexity	Range of Reading and Level of Complexity
10. Actively engage in shared reading	<b>10.</b> With prompting and support, read prose	<b>10.</b> By the end of the year, read and
activities using literature from a variety of	and poetry from a variety of cultures of	comprehend a range of literature from a
cultures with purpose and understanding,	appropriate complexity for grade 1.	variety of cultures, within a complexity band
and scaffolding as needed.		appropriate to grade 2 (from upper grade 1 to
		grade 3), with scaffolding as needed at the
		high end of the range.

# **Reading Standards for Literature K-5**

Grade 3 students:	Grade 4 students:	Grade 5 students:
Key Ideas and Details	Key Ideas and Details	Key Ideas and Details
<ol> <li>Ask and answer questions to demonstrate understanding of a text (e.g., making basic inferences and predictions), referring explicitly to details from the text as the basis for the answers.</li> <li>Determine the author's purpose, message, lesson, or moral and explain how it is conveyed through key details in the text; summarize stories in correct sequence, including fables and folktales from diverse cultures.</li> </ol>	<ol> <li>Refer to details and examples in a text when explaining what the text says explicitly and when drawing inferences from the text.</li> <li>Determine a theme or author's message or purpose of a story, drama, or poem using details and evidence from the text as support; summarize main ideas or events, in correct sequence, including how conflicts are resolved.</li> </ol>	<ol> <li>Locate explicit information in the text to support inferences drawn from the text.</li> <li>Quote accurately from a text when explaining what the text says explicitly and when drawing inferences from the text.</li> <li>Determine a theme or author's message or purpose of a story, drama, or poem using details and evidence from the text as support, including how characters in a story or drama respond to challenges or how the speaker in a poem reflects upon a topic; summarize main ideas or events, in correct</li> </ol>
3. Describe characters in a story (e.g., their traits, motivations, or feelings) and explain how their actions contribute to the sequence of events (e.g., creating or solving a problem).	3. Describe in depth a character, setting, or event in a story or drama, drawing on specific details in the text (e.g., a character's thoughts, words, or actions).  Craft and Structure	sequence.  3. Compare and contrast two or more characters, settings, or events in a story or drama, drawing on specific details in the text (e.g., how characters interact, how conflicts are resolved).  Craft and Structure
Craft and Structure		
<b>4.</b> Determine the meaning of words and phrases as they are used in a text, distinguishing literal meanings from use of figurative language (e.g., exaggeration in tall tales).	<b>4.</b> Determine the meaning of words and phrases as they are used in a text, including use of figurative language and literary devices (e.g., imagery, metaphors, similes).	<b>4.</b> Determine the meaning of words and phrases as they are used in a text, including use of figurative language and literary devices (e.g., imagery, metaphors, analogies, hyperbole).
<b>5.</b> Refer to parts of stories, dramas, and poems when writing or speaking about a text, using terms such as chapter, scene, and stanza; describe how each successive part builds on earlier sections.	<b>5.</b> Explain major differences between poems, drama, and prose, and refer to the structural elements of poems (e.g., verse and rhythm) and drama (e.g., casts of characters, settings, descriptions, dialogue, stage directions) when writing or speaking about a text.	<b>5.</b> Explain how a series of chapters, scenes, or stanzas fits together to provide the overall structure of a particular story, drama, or poem.
<b>6.</b> Distinguish their own point of view from that of the narrator or those of the	<b>6.</b> Compare and contrast the point of view from which different stories are narrated,	<b>6.</b> Describe how a narrator's or speaker's point of view (e.g., first person, third person)

Grade 3 students:	Grade 4 students:	Grade 5 students:
characters.	including how the use of first or third person	influences how events are described or how
	can change the way a reader might see	characters are developed and portrayed.
	characters or events described.	
Integration of Knowledge and Ideas	Integration of Knowledge and Ideas	Integration of Knowledge and Ideas
7. Explain how specific aspects of text's	7. Make connections between the text of a	7. Analyze how visual and multimedia
illustrations contribute to what is	story or drama and a visual or oral	elements contribute to the meaning, tone, or
conveyed by the words in a story (e.g.,	presentation of the text, identifying where	personal appeal of a text (e.g., graphic novel,
create mood, emphasize aspects of a	each version reflects specific descriptions	multimedia presentation of fiction, folktale,
character or setting).	and directions in the text.	myth, poem).
8. (Not applicable to literature)	8. (Not applicable to literature)	8. (Not applicable to literature)
9. Compare and contrast the themes,	9. Compare and contrast the treatment of	9. Compare and contrast stories in the same
settings, and plots of stories written by	similar themes and plots (e.g., opposition of	genre (e.g., mysteries and adventure stories)
the same author about the same or	good and evil) and patterns of events (e.g.,	on their approaches to similar themes and
similar characters (e.g., in books from a	the quest) in literature.	plot development.
series).		
Range of Reading and Level of Text	Range of Reading and Level of Text	Range of Reading and Level of Text
Complexity	Complexity	Complexity
<b>10.</b> By the end of the year, read and	<b>10.</b> By the end of the year, read and	<b>10.</b> By the end of the year, read and
comprehend a range of literature from a	comprehend a range of literature from a	comprehend a range of literature from a
variety of cultures, within a complexity band	variety of cultures, within a complexity	variety of cultures, within a complexity band
appropriate to grade 3 (from upper grade 2 to	band appropriate to grade 4 (from upper	appropriate to grade 5 (from upper grade 4 to
grade 4), with scaffolding as needed at the	grade 3 to grade 5), with scaffolding as	grade 6), with scaffolding as needed at the
high end of the range.	needed at the high end of the range.	high end of the range.

# **Reading Standards for Informational Text K-5**

Kindergartners:	Grade 1 students:	Grade 2 students:
Key Ideas and Details	Key Ideas and Details	Key Ideas and Details
1. With prompting and support, elicit background/prior knowledge and experience in order to ask and answer questions about an informational text using key details from the	1. With prompting and support, elicit background/prior knowledge and experience in order to ask and answer questions about an informational text using key details from	<b>1.</b> Ask and answer such questions as who, what, where, when, why, and how to demonstrate understanding of informational texts using key details from the text.
text.	the text.	
<b>2.</b> With prompting and support, identify the main topic and retell key details of a text.	2. Identify the main topic or author's purpose (e.g., to teach or tell us about) and retell key details of a text.	<b>2.</b> Identify the main topic of a multiparagraph text as well as the focus of specific paragraphs within the text.
<b>3.</b> With prompting and support, describe the connection between two individuals, events, ideas, or pieces of information in a text.	<b>3.</b> Describe the connection between two individuals, events, ideas, or pieces of information in a text.	<b>3.</b> Describe the connection between a series of historical events, scientific ideas or concepts, or steps in technical procedures in a text.
Craft and Structure	Craft and Structure	Craft and Structure
<b>4.</b> With prompting and support, ask and answer questions about unknown words in a text.	<b>4.</b> Ask and answer questions to help determine or clarify the meaning of words and phrases in a text.	<b>4.</b> Determine the meaning of words and phrases in a text relevant to a grade 2 topic or subject area.
<b>5.</b> Identify the front cover, back cover, and title page of a book.	<b>5.</b> Know and use various text features (e.g., title, labels with graphics, bold print, visual cues such as arrows, electronic menus, icons) to locate key facts or information in a text.	<b>5.</b> Know and use various text features (e.g., captions, bold print, headings, charts, bulleted or numbered lists, electronic menus, icons) to locate key facts or information in a text efficiently.
<b>6.</b> Name the author and illustrator of a text and describe the role of each in presenting the ideas or information in a text.	<b>6.</b> Distinguish between information provided by photos or other graphics and information provided by the words in a text.	<b>6.</b> Identify the main purpose of a text, including what the author wants to answer, explain, or describe.
Integration of Knowledge and Ideas	Integration of Knowledge and Ideas	Integration of Knowledge and Ideas
<b>7.</b> With prompting and support, describe the relationship between illustrations and the text in which they appear (e.g., what person, place, thing, or idea in the text an illustration depicts).	<b>7.</b> Use the illustrations and details in a text to describe its key ideas.	<b>7.</b> Explain how specific images (e.g., a diagram showing how a machine works) contribute to and clarify a text.

Kindergartners:	Grade 1 students:	Grade 2 students:
8. With prompting and support, identify the	8. Identify the opinions an author states to	8. Describe how reasons given support
opinions an author states in a text.	support points in a text.	specific opinions the author states in a text.
<b>9.</b> With prompting and support, identify basic	9. Identify basic similarities in and	<b>9.</b> Compare and contrast the most important
similarities in and differences between	differences between information presented	points presented by two texts or related
information presented in two texts on the	in two texts on the same topic (e.g., compare	topics (e.g., a book about polar bears and a
same topic (e.g., compare two photos or	two graphics, descriptions, or steps in a	book about black bears).
diagrams, compare two animal babies).	process to make something).	
Range of Reading and Level of Text	Range of Reading and Level of Text	Range of Reading and Level of Text
Complexity	Complexity	Complexity
10. Actively engage in shared reading	<b>10.</b> With prompting and support, read	<b>10.</b> By the end of the year, read and
activities using a range of topics and texts	informational texts on a range of topics	comprehend a range of informational texts,
with purpose and understanding, with	appropriately complex for grade 1, with	including history/social studies, science, and
scaffolding as needed.	scaffolding as needed.	technical texts within a complexity band
		appropriate to grade 2 (from upper grade 1 to
		grade 3), with scaffolding as needed at the
		high end of the range.

# Reading Standards for Informational Text K-5

Grade 3 students:	Grade 4 students:	Grade 5 students:
Key Ideas and Details	Key Ideas and Details	Key Ideas and Details
1. Ask and answer questions to demonstrate understanding of a text, (e.g., explaining what the texts says explicitly, making basic inferences and predictions), referring explicitly to the text as the basis for the answers.	1. Locate explicit information in the text to explain what the text says explicitly and to support inferences drawn from the text.	1. Locate explicit information in the text to explain what the text says explicitly and to support inferences drawn from the text.
2. Determine the main idea of a text and locate details that support the main idea; paraphrase or summarize main ideas or events in a multi-paragraph text, including correct sequence and details that support the main idea.	2. Determine the main idea of a text and explain how it is supported by key details; paraphrase or summarize key ideas, events, or procedures including correct sequence when appropriate.	2. Determine the main idea and subtopics of a text and explain how they are supported by key details; paraphrase or summarize key ideas, events, or procedures including correct sequence when appropriate.
<b>3.</b> Describe the relationship or connection among a series of historical events, scientific ideas or concepts, or steps in technical procedures in a text, using language that pertains to time, sequence, and cause/effect.	<b>3.</b> Explain relationships (e.g., cause-effect) among events, procedures, ideas, or concepts in a historical, scientific, or technical text, including what happened and why, based on specific information in the text.	<b>3.</b> Explain the relationships (e.g., cause-effect) or interactions among two or more individuals, events, ideas, or concepts in a historical, scientific, or technical text based on specific information in the text.
Craft and Structure	Craft and Structure	Craft and Structure
<b>4.</b> Determine the meaning of general academic and domain-specific words and phrases in a text relevant to a grade 3 topic or subject area.	<b>4.</b> Determine the meaning of general academic and domain-specific words or phrases in a text relevant to a grade 4 topic or subject area.	<b>4.</b> Determine the meaning of general academic and domain-specific words and phrases in a text relevant to a grade 5 topic or subject area.
<ul> <li>5. Use text features and search tools (e.g., table of contents, index, key words, sidebars, hyperlinks) to locate information relevant to a given topic efficiently.</li> <li>6. Determine author's purpose; distinguish</li> </ul>	<ul> <li>5. Describe the overall structure (e.g., sequence, comparison, cause/effect, problem/solution) of events, ideas, concepts, or information in a text or part of a text.</li> <li>6. Determine author's purpose; compare and</li> </ul>	<ul> <li>5. Compare and contrast the overall structure (e.g., sequence, comparison, cause/effect, problem/solution) of events, ideas, concepts, or information in two or more texts.</li> <li>6. Determine author's purpose; analyze</li> </ul>
own point of view from that of the author of a text.	contrast a firsthand and secondhand account of the same event or topic; describe the differences in focus and the information provided.	multiple accounts of the same event or topic, noting important similarities and differences in the points of view they represent. (e.g.,

Grade 3 students:	Grade 4 students:	Grade 5 students:
		social studies topics, media messages about
		current events).
Integration of Knowledge and Ideas	Integration of Knowledge and Ideas	Integration of Knowledge and Ideas
7. Use information gained from illustrations	7. Interpret information presented visually,	7. Draw on information from multiple print or
(e.g., maps, photographs), and the words in a	orally, or quantitatively (e.g., in charts,	digital sources, demonstrating the ability to
text to demonstrate understanding of the text	graphs, diagrams, time lines, animations, or	locate an answer to a question quickly or to
(e.g., where, when, why, and how key events	interactive elements on Web pages) and	solve a problem efficiently.
occur).	explain how the information contributes to	
	an understanding of the text in which it	
	appears.	
8. Describe the logical connection between	<b>8.</b> Explain how an author uses reasons and	8. Explain how an author uses reasons and
particular sentences and paragraphs in a text	evidence to support particular points in a	evidence to support particular points in a text,
(e.g., comparison, cause/effect,	text.	identifying which reasons and evidence
first/second/third in a sequence).		supports which point(s).
<b>9.</b> Compare and contrast the most important	<b>9.</b> Integrate information from two texts on	<b>9.</b> Integrate information from several texts on
points and key details presented in two texts	the same topic or related topics in order to	the same topic or related topics in order to
on the same topic or related topics.	write or speak about the subject	write or speak about the subject
	knowledgeably.	knowledgeably.
Range of Reading and Level of Text	Range of Reading and Level of Text	Range of Reading and Level of Text
Complexity	Complexity	Complexity
<b>10.</b> By the end of the year, read and	<b>10.</b> By the end of the year, read and	<b>10.</b> By the end of the year, read and
comprehend a range of informational texts,	comprehend a range of informational texts,	comprehend a range of informational texts,
including history/social studies, science, and	including history/social studies, science, and	including history/social studies, science, and
technical texts within a complexity band	technical texts within a complexity band	technical texts within a complexity band
appropriate to grade 3 (from upper grade 2 to	appropriate to grade 4 (from upper grade 3	appropriate to grade 5 (from upper grade 4 to
grade 4), with scaffolding as needed at the	to grade 5), with scaffolding as needed at the	grade 6), with scaffolding as needed at the
high end of the range.	high end of the range.	high end of the range.

# **Reading Standards: Foundational Skills K-5**

These standards are directed toward fostering students' understanding and working knowledge of concepts of print, the alphabetic principle, and other basic conventions of the English writing system. These foundational skills are not an end in and of themselves; rather, they are necessary and important components of an effective, comprehensive reading program designed to develop proficient readers with the capacity to comprehend texts across a range of types and disciplines. Instruction should be differentiated: good readers will need much less practice with these concepts than struggling readers will. The point is to teach students what they need to learn and not what they already know—to discern when particular children or activities warrant more or less attention.

Note: In kindergarten, children are expected to demonstrate increasing awareness and competence in the areas that follow.

Kindergartners:	Grade 1 students:
Print Concepts	Print Concepts
1. Demonstrate understanding of the organization and basic features of	1. Demonstrate understanding of the organization and basic
print.	features of print.
a. Follow words from left to right, top to bottom, and page-by-page.	a. Recognize the distinguishing features of a sentence (e.g., first
b. Recognize that spoken words are represented in written language by	word, capitalization, ending punctuation).
specific sequences of letters.	
c. Understand that words are separated by spaces in print.	
d. Recognize and name all upper- and lowercase letters of the	
alphabet.	
Phonological Awareness	Phonological Awareness
2. Demonstrate understanding of spoken `words, syllables, and sounds	2. Demonstrate understanding of spoken words, syllables, and
(phonemes).	sounds (phonemes).
a. Recognize and produce rhyming words.	a. Distinguish long from short vowel sounds in spoken single-syllable
b. Count, pronounce, blend, and segment syllables in spoken words.	words.
c. Blend and segment onsets and rimes of single-syllable spoken words.	b. Orally produce single-syllable words by blending sounds
d. Isolate and pronounce the initial, medial vowel, and final sounds	(phonemes), including consonant blends.
(phonemes) in three-phoneme (consonant-vowel-consonant, or CVC)	c. Isolate and pronounce initial, medial vowel, and final sounds
words.* (This does not include CVCs ending with /l/, /r/, or /x/.)	(phonemes) in spoken single-syllable words.
e. Add or substitute individual sounds (phonemes) in simple, one-	d. Segment spoken single-syllable words into their complete
syllable words to make new words.	sequence of individual sounds (phonemes).
*Words, syllables, or phonemes written in /slashes/refer to their	
pronunciation or phonology. Thus, /CVC/ is a word with three	
phonemes regardless of the number of letters in the spelling of the word.	

# Reading Standards: Foundational Skills K-5

Note: In kindergarten, children are expected to demonstrate increasing awareness and competence in the areas that follow.

Kindergartners:	Grade 1 students:	Grade 2 students:
Phonics and Word Recognition	Phonics and Word Recognition	Phonics and Word Recognition
3. Know and apply grade-level phonics and	3. Know and apply grade-level phonics and	3. Know and apply grade-level phonics and
word analysis skills in decoding words.	word analysis skills in decoding words.	word analysis skills in decoding words.
a. Demonstrate basic knowledge of one-to-	a. Know the spelling-sound correspondences	a. Distinguish long and short vowels when
one letter-sound correspondences by	for common consonant digraphs.	reading regularly spelled one-syllable words.
producing the primary sound or many of the	b. Decode regularly spelled one-syllable	b. Know spelling-sound correspondences for
most frequent sounds for each consonant.	words.	additional common vowel teams.
b. Associate the long and short sounds with	c. Know final -e and common vowel team	c. Decode regularly spelled two-syllable
the common spellings (graphemes) for the	conventions for representing long vowel	words with long vowels.
five major vowels.	sounds.	d. Decode words with common prefixes and
c. Read common high-frequency words by	d. Use knowledge that every syllable must	suffixes.
sight. (e.g., the, of, to, you, she, my, is, are,	have a vowel sound to determine the number	e. Identify words with inconsistent but
do, does).	of syllables in a printed word.	common spelling-sound correspondences.
d. Distinguish between similarly spelled	e. Decode two-syllable words following basic	f. Recognize and read grade-appropriate
words by identifying the sounds of the letters	patterns by breaking the words into syllables.	irregularly spelled words.
that differ.	f. Read words with inflectional endings.	
	g. Recognize and read grade-appropriate	
	irregularly spelled words.	
Fluency	Fluency	Fluency
<b>4.</b> Read emergent-reader texts with purpose	<b>4.</b> Read with sufficient accuracy and fluency	<b>4.</b> Read with sufficient accuracy and fluency
and understanding.	to support comprehension.	to support comprehension.
	a. Read on-level text with purpose and	a. Read on-level text with purpose and
	understanding.	understanding.
	b. Read on-level text orally with accuracy,	b. Read on-level text orally with accuracy,
	appropriate rate, and expression on	appropriate rate, and expression on
	successive readings.	successive readings.
	c. Use context to confirm or self-correct word	c. Use context to confirm or self-correct word
	recognition and understanding, rereading as	recognition and understanding, rereading as
	necessary.	necessary.

# Reading Standards: Foundational Skills K-5

Grade 3 students:	Grade 4 students:	Grade 5 students:
Phonics and Word Recognition*	Phonics and Word Recognition*	Phonics and Word Recognition*
3. Know and apply grade-level phonics and	3. Know and apply grade-level phonics and	3. Know and apply grade-level phonics and
word analysis skills in decoding words.	word analysis skills in decoding words.	word analysis skills in decoding words.
a. Identify and know the meaning of the most	a. Use combined knowledge of all letter-sound	a. Use combined knowledge of all letter-
common prefixes and derivational suffixes.	correspondences, syllabication patterns, and	sound correspondences, syllabication
b. Decode words with common Latin suffixes.	morphology (e.g., roots and affixes) to read	patterns, and morphology (e.g., roots and
c. Decode multisyllable words.	accurately unfamiliar multisyllabic words in	affixes) to read accurately unfamiliar
d. Read grade-appropriate irregularly spelled	context and out of context.	multisyllabic words in context and out of
words.		context.
Fluency	Fluency	Fluency
4. Read with sufficient accuracy and fluency	<b>4.</b> Read with sufficient accuracy and fluency to	<b>4.</b> Read with sufficient accuracy and fluency
to support comprehension.	support comprehension.	to support comprehension.
a. Read on-level text with purpose and	a. Read on-level text with purpose and	a. Read on-level text with purpose and
understanding.	understanding.	understanding.
b. Read on-level prose and poetry orally with	b. Read on-level prose and poetry orally with	b. Read on-level prose and poetry orally with
accuracy, appropriate rate, and expression on	accuracy, appropriate rate, and expression on	accuracy, appropriate rate, and expression
successive readings.	successive readings.	on successive readings.
c. Use context to confirm or self-correct word	c. Use context to confirm or self-correct word	c. Use context to confirm or self-correct word
recognition and understanding, rereading as	recognition and understanding, rereading as	recognition and understanding, rereading as
necessary.	necessary.	necessary.

<sup>\*</sup>There are no foundational skills 1 and 2 for grades 3-5.

The following standards for K - 5 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. Each year in their writing, students should demonstrate increasing sophistication in all aspects of language use, from vocabulary and syntax to the development and organization of ideas, and they should address increasingly demanding content and sources. Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.

Kindergartners:	Grade 1 students:	Grade 2 students:
Text Types and Purposes	Text Types and Purposes	Text Types and Purposes
1. Use a combination of drawing, dictating,	1. Write opinion pieces in which they	1. Write opinion pieces in which they
and writing to state an opinion or a	introduce the topic or name the book they	introduce the topic or book they are writing
preference about a topic or part of a book	are writing about, state an opinion or	about, state an opinion, supply reasons that
(e.g., I like dogs better than cats because;	preference, supply a fact (e.g., because race	support the opinion, use linking words (e.g.,
My favorite part of the story is when; I think	cars go faster than) or reason for the	because, and, also) to connect opinion and
it was funny when).	opinion, and end with a sentence that	reasons, and provide one or more concluding
	restates their opinion related to a feeling or	sentences that restate or paraphrase their
	emotion (e.g., it makes me laugh; that was	opinion.
	the scariest part).	
2. Use a combination of drawing, dictating,	2. Write informative/explanatory texts in	2. Write informative/explanatory texts in
and writing to compose	which they name a topic, supply some facts	which they introduce a topic, use facts and
informative/explanatory texts in which they	about the topic, and provide some sense of	definitions to develop points, and provide a
name what they are writing about and supply	closure (e.g., restate at the end the most	concluding statement or one or more
some information about the topic.	interesting fact or the most important idea	concluding sentences that emphasize their
	shared).	most important point or focus.
<b>3.</b> Use a combination of drawing, dictating,	3. Use narrative writing to recount two or	3. Use narrative writing to retell a well-
and writing to narrate a real or imagined	more real or imagined sequenced events,	elaborated event or short sequence of real
event or several loosely linked events, tell	include some details regarding what	or imagined events, include details to
about the events in the order in which they	happened, who was there, use linking words	describe actions, thoughts, and feelings, use
occurred, and provide an ending (how the	to signal event order (e.g., first, next, then),	linking words to signal event order, and
problem was solved) or a reaction (e.g., a	and provide an ending (e.g., how the problem	provide one or more concluding sentences
feeling) to what happened.	was solved; how someone felt at the end).	that restate or emphasize a feeling or lesson
		learned.
Production and Distribution of Writing	Production and Distribution of Writing	Production and Distribution of Writing
4. (Begins in grade 3.)	4. (Begins in grade 3)	<b>4.</b> (Begins in grade 3)

Kindergartners:	Grade 1 students:	Grade 2 students:
5. With guidance and support from adults,	5. With guidance and support from adults,	5. With guidance and support from adults
respond to questions and suggestions from	focus on a topic, respond to questions and	and peers, focus on a topic and strengthen
peers and add details to strengthen writing as	suggestions from peers, and add concrete	writing as needed (e.g., adding concrete and
needed.	and sensory details to strengthen writing as	sensory details; elaborating on how the
	needed.	details chosen support the focus) by revising
		and editing.
<b>6.</b> With guidance and support from adults,	<b>6.</b> With guidance and support from adults,	<b>6.</b> With guidance and support from adults,
explore a variety of digital tools to produce	use a variety of digital tools to produce and	use a variety of digital tools to produce and
and publish writing, including in collaboration	publish writing, including in collaboration	publish writing, including in collaboration
with peers.	with peers.	with peers.
Research to Build and Present Knowledge	Research to Build and Present Knowledge	Research to Build and Present Knowledge
7. Participate in shared research and writing	7. Participate in shared research and writing	7. Participate in shared research and writing
projects (e.g., explore a number of books by a	projects (e.g., explore a number of "how-to"	projects (e.g., read a number of books on a
favorite author and combine or summarize	books on a given topic and use them to write	single topic to produce a report or visual or
information/facts learned or express	a sequence of instructions or combine or	oral presentation; record data from science
opinions about them).	summarize information/facts learned).	observations).
8. With guidance and support from adults,	8. With guidance and support from adults,	8. Recall information from experiences or
recall information from experiences or gather	recall information from experiences or	gather information from provided sources to
information from provided sources to answer	gather information from provided sources to	answer a question.
a question.	answer a question.	
9. (Begins in grade 4)	9. (Begins in grade 4)	9. (Begins in grade 4)
Range of Writing	Range of Writing	Range of Writing
<b>10.</b> (Begins in grade 3)	<b>10.</b> (Begins in grade 3)	<b>10.</b> (Begins in grade 3)

Grade 3 students:	Grade 4 students:	Grade 5 students:
Text Types and Purposes	Text Types and Purposes	Text Types and Purposes
1. Write opinion pieces on topics or texts, supporting a point of view with fact- or text-based reasons (e.g., I like large dogs better than small dogs because they can pull a sled and run for a longer time than small dogs can).  a. Introduce the topic or text they are writing about, state an opinion, and create an organizational structure that lists fact- or text-based reasons.  b. Provide reasons that support the opinion. c. Link opinion and reasons using words and phrases (e.g., because, therefore, since, for example).	1. Write opinion pieces on topics or texts, supporting a point of view with fact- or text-based reasons and information (e.g., The character was brave because she). a. Introduce a topic or text clearly, state an opinion, and create an organizational structure in which related ideas are grouped to support the writer's purpose. b. Provide reasons that are supported by facts and details. c. Link opinion and reasons using words and phrases (e.g., for instance, in order to, in addition). d. Provide a concluding statement or section	1. Write opinion pieces on topics or texts, supporting a point of view with fact- or text-based reasons and information. a. Introduce a topic or text clearly, state an opinion, and create an organizational structure in which ideas are logically grouped to support the writer's purpose. b. Provide logically ordered reasons that are supported by facts and details. c. Link opinion and reasons using words, phrases, and clauses (e.g., consequently, specifically, most of all). d. Provide a concluding statement or section that reinforces or restates the opinion
<ul> <li>d. Provide a concluding statement or section that reinforces or restates the opinion.</li> <li>2. Write informative/explanatory texts to examine a topic and convey ideas and information clearly.</li> </ul>	that reinforces or restates the opinion presented.  2. Write informative/explanatory texts to examine a topic and convey ideas and information clearly.	<ul><li>presented.</li><li>2. Write informative/explanatory texts to examine a topic and convey ideas and information clearly.</li></ul>
<ul> <li>a. Introduce a topic and group related information together; include labeled or captioned visuals when useful to aiding comprehension.</li> <li>b. Develop the topic with facts, definitions, details, and explanations that support the focus.</li> <li>c. Use linking words and phrases (e.g., also, another, and, more, but) to connect ideas within categories of information.</li> </ul>	a. Introduce a topic clearly and group related information in paragraphs and sections; include formatting (e.g., headings), labeled or captioned visuals/diagrams, and multimedia when useful to aiding comprehension. b. Develop the topic with facts, definitions, concrete details, quotations, or other information/explanations and examples that support the focus. c. Link ideas within categories of information using words and phrases (e.g., another, for example, also, because).	a. Introduce a topic clearly, provide a general observation and focus, and group related information logically; include formatting (e.g., headings), labeled or captioned visuals/graphics, and multimedia when useful to aiding comprehension. b. Develop the topic with facts, definitions, concrete details, quotations, or other information and examples that support the topic. c. Link ideas within and across categories of information using words, phrases, and clauses (e.g., in contrast, especially).

Grade 3 students:	Grade 4 students:	Grade 5 students:
d. Provide a concluding statement or section	d. Use precise language and domain-specific	d. Use precise language and domain-specific
that paraphrases the focus of the text.	vocabulary to inform about or explain the	vocabulary to inform about or explain the
	topic.	topic.
	e. Provide a concluding statement or section	e. Provide a concluding statement or section
	that paraphrases the focus of the text or	that paraphrases the focus of the text or
	explanation presented.	explanation presented.
<b>3.</b> Use narrative writing to develop real or	<b>3.</b> Use narrative writing to develop real or	3. Use narrative writing to develop real or
imagined characters, experiences, or events	imagined characters, experiences, or events	imagined characters, experiences, or events
using effective narrative techniques	using effective narrative techniques	using effective narrative techniques (dialogue,
(dialogue, description, elaboration, problem-	(dialogue, description, elaboration, problem-	description, elaboration, problem-solution,
solution, figurative language), and clear event	solution, figurative language) and clear event	figurative language) and clear event
sequences (chronology).	sequences (chronology).	sequences (chronology).
a. Establish a context or situation and	a. Orient the reader by establishing a context	a. Orient the reader by establishing a context
introduce a narrator and/or characters;	or situation and introducing a narrator	or situation and introducing a narrator and/or
organize an event sequence that unfolds	and/or characters; organize an event	characters; organize an event sequence that
naturally.	sequence that unfolds naturally.	unfolds naturally.
b. Use narrative techniques, such as dialogue,	b. Use narrative techniques, such as dialogue,	b. Use narrative techniques, such as dialogue,
description and elaboration, and concrete	description and elaboration, and concrete	description and elaboration, concrete and
and sensory details to describe actions,	and sensory details to describe actions,	sensory details, literary devices, and pacing to
thoughts, and feelings and to develop	thoughts, and motivations and to develop	describe actions, thoughts, and motivations
experiences and events showing the	experiences and events showing the	and to develop experiences and events
response of characters to situations or	responses of characters to situations,	showing the responses of characters to
problems.	problems, or conflicts.	situations, problems, or conflicts.
c. Use transitional words and phrases to	c. Use a variety of transitional words and	c. Use a variety of transitional words, phrases,
signal event sequences (e.g., later, soon	phrases to develop the sequence of events.	and devices (e.g., foreshadowing) to develop
after).	d. Use concrete words and phrases and	the pacing and sequence of events.
d. Provide a sense of closure (e.g., how a	sensory details, and elaboration to convey	d. Use concrete words and phrases, sensory
problem was solved or how the event	experiences and events precisely.	details, and elaboration to convey experiences
ended).	e. Provide a conclusion that follows from the	and events precisely.
	narrated experiences or events.	e. Provide a conclusion that follows from the
		narrated experiences or events.

Grade 3 students:	Grade 4 students:	Grade 5 students:
Production and Distribution of Writing	Production and Distribution of Writing	Production and Distribution of Writing
<b>4.</b> With guidance and support from adults,	4. Produce clear and coherent writing in	4. Produce clear and coherent writing in
produce writing in which the development	which the development and organization are	which the development and organization are
and organization are appropriate to task and	appropriate to task, purpose, and audience.	appropriate to task, purpose, and audience.
purpose. (Grade-specific expectations for	(Grade-specific expectations for writing types	(Grade-specific expectations for writing types
writing types are defined in standards 1–3	are defined in standards 1–3 above.)	are defined in standards 1–3 above.)
above.)		
<b>5.</b> With guidance and support from peers and	5. With guidance and support from peers and	5. With guidance and support from peers and
adults, develop and strengthen writing as	adults, develop and strengthen writing as	adults, develop and strengthen writing as
needed by planning, revising, and editing.	needed by planning, revising, and editing.	needed by planning, revising, editing,
(Editing for conventions should demonstrate	(Editing for conventions should demonstrate	rewriting, or trying a new approach. (Editing
command of Language standards 1–3 up to	command of Language standards 1–3 up to	for conventions should demonstrate
and including grade 3.)	and including grade 4.)	command of Language standards 1–3 up to
		and including grade 5.)
<b>6.</b> With guidance and support from adults,	<b>6.</b> With some guidance and support from	<b>6.</b> With some guidance and support from
use technology to produce and publish	adults, use technology, including the	adults, use technology, including the Internet,
writing (using keyboarding skills) as well as to	Internet, to produce, edit, and publish	to produce, edit, and publish writing as well as
interact and collaborate with others and to	writing as well as to interact and collaborate	to interact and collaborate with others;
locate information about topics.	with others and to locate information about	demonstrate sufficient command of
	topics; demonstrate sufficient command of	keyboarding skills to type a minimum of two
	keyboarding skills to type a minimum of one	pages in a single sitting.
	page in a single sitting.	
Research to Build and Present Knowledge	Research to Build and Present Knowledge	Research to Build and Present Knowledge
7. Conduct short research projects that build	7. Conduct short research projects that build	7. Conduct short research projects that use
knowledge about a topic.	knowledge through investigation of different	several sources to build knowledge through
	aspects of a topic.	investigation of different aspects of a topic.
<b>8.</b> Recall information from experiences or	8. Recall relevant information from	8. Recall relevant information from
gather information from print and digital	experiences or gather relevant information	experiences or gather relevant information
sources; take brief notes on sources and sort	from print and digital sources; take notes and	from print and digital sources; summarize or
evidence into provided categories.	categorize information, and provide a list of	paraphrase information in notes and finished
	sources.	work, and provide a list of sources.

### Writing Standards K-5

Grade 3 students:	Grade 4 students:	Grade 5 students:
9. (Begins in grade 4)	9. Draw evidence from literary or	9. Draw evidence from literary or
	informational texts to support analysis,	informational texts to support analysis,
	reflection, and research.	reflection, and research.
	a. Apply grade 4 Reading standards to	a. Apply grade 5 Reading standards to
	literature (e.g., "Describe in depth a	literature (e.g., "Compare and contrast two or
	character, setting, or event in a story or	more characters, settings, or events in a story
	drama, drawing on specific details in the text	or drama, drawing on specific details in the
	[e.g., a character's thoughts, words, or	text [e.g., how characters interact, how
	actions].").	conflicts are resolved].").
	b. Apply grade 4 Reading standards to	b. Apply grade 5 Reading standards to
	informational texts (e.g., "Explain how an	informational texts (e.g., "Explain how an
	author uses reasons and evidence to support	author uses reasons and evidence to support
	particular points in a text.").	particular points in a text, identifying which
		reasons and evidence supports which
		point[s].").
Range of Writing	Range of Writing	Range of Writing
<b>10.</b> Write routinely over extended time	<b>10.</b> Write routinely over extended time	<b>10.</b> Write routinely over extended time
frames (time for research, reflection, and	frames (time for research, reflection, and	frames (time for research, reflection, and
revision) and shorter time frames (a single	revision) and shorter time frames (a single	revision) and shorter time frames (a single
sitting or a day or two) for a range of	sitting or a day or two) for a range of	sitting or a day or two) for a range of
discipline-specific tasks, purposes, and	discipline-specific tasks, purposes, and	discipline-specific tasks, purposes, and
audiences.	audiences.	audiences.

## **Speaking and Listening Standards K-5**

The following standards for K - 5 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.

Kindergartners:	Grade 1 students:	Grade 2 students:
Comprehension and Collaboration	Comprehension and Collaboration	Comprehension and Collaboration
1. Participate in collaborative conversations	1. Participate in collaborative conversations	1. Participate in collaborative conversations
with diverse partners about kindergarten	with diverse partners about grade 1 topics	with diverse partners about grade 2 topics
topics and texts with peers and adults in small	and texts with peers and adults in small and	and texts with peers and adults in small and
and larger groups.	larger groups.	larger groups.
a. Follow agreed-upon rules for discussions	a. Follow agreed-upon rules for discussions	a. Follow agreed-upon rules for discussions
(e.g., listening to others and taking turns	(e.g., listening to others with care, speaking	(e.g., gaining the floor in respectful ways,
speaking about the topics and texts under	one at a time about the topics and texts	listening to others with care, speaking one at
discussion).	under discussion).	a time about the topics and texts under
b. Continue a conversation through multiple	b. Build on others' talk in conversations by	discussion).
exchanges.	responding to the comments of others	b. Build on others' talk in conversations by
	through multiple exchanges.	linking their comments to the remarks of
	c. Ask questions to clear up any confusion	others.
	about the topics and texts under discussion.	c. Ask for clarification and further explanation
		as needed about the topics and texts under
2. Cariffers and and and and all and and and and and	2 Advantage and addate	discussions.
2. Confirm understanding of a text read aloud	2. Ask and answer questions about key details	2. Retell or describe key ideas or details from
or information presented orally or through	in a text read aloud or information presented	a text read aloud or information presented
other media by asking and answering	orally or through other media.	orally or through other media.
questions about key details and requesting		
clarification if something is not understood.	2 Ash and annual resident should be a	2 Ash and annual suppliers should that a
<b>3.</b> Ask and answer questions in order to seek	3. Ask and answer questions about what a	<b>3.</b> Ask and answer questions about what a
help, get information, or clarify something that is not understood.	speaker says in order to gather additional	speaker says in order to clarify
triat is not understood.	information or clarify something that is not understood.	comprehension, gather additional
	understood.	information, or deepen understanding of a
Presentation of Knowledge and Ideas	Presentation of Knowledge and Ideas	topic or issue.  Presentation of Knowledge and Ideas
i resentation of knowledge and lacas	i resentation of knowledge and facas	i resentation of knowledge and lacas

Kindergartners:	Grade 1 students:	Grade 2 students:
<b>4.</b> Describe familiar people, places, things, and events, and with prompting and support, provide additional related details.	<b>4.</b> Describe people, places, things, and events with relevant details, expressing ideas and feelings clearly.	<b>4.</b> Tell a story or retell an experience with relevant facts and relevant, descriptive details, speaking audibly in coherent sentences.
<b>5.</b> Add drawings or other visual displays to descriptions as desired to provide additional details.	<b>5.</b> Add drawings or other visual displays to descriptions when appropriate to clarify ideas, thoughts, and feelings.	<b>5.</b> Create audio recordings of stories or poems; add drawings or other visual displays to stories or recounts of experiences when appropriate to clarify ideas, thoughts, and feelings.
<b>6.</b> Speak audibly and express thoughts, feelings, and ideas clearly in complete sentences when appropriate to task and situation.	6. Produce complete sentences when appropriate to task and situation. (See grade 1 Language standards 1 and 3 for specific expectations.)	<b>6.</b> Produce complete sentences when appropriate to task and situation in order to provide requested detail or clarification. (See grade 2 Language standards 1 and 3 for specific expectations.)

### **Speaking and Listening Standards K-5**

Grade 3 students:	Grade 4 students:	Grade 5 students:
Comprehension and Collaboration	Comprehension and Collaboration	Comprehension and Collaboration
Comprehension and Collaboration  1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 3 topics and texts, building on others' ideas and expressing their own clearly.  a. After learning a protocol (e.g., Socratic method), come to discussions prepared, having read or studied required material; explicitly draw on that preparation and other information known about the topic to explore ideas under discussion.  b. Follow agreed-upon rules for discussions (e.g., gaining the floor in respectful ways, listening to others with care, speaking one at a time about the topics and texts under discussion).  c. Ask questions to check understanding of information presented, stay on topic, and link their comments to the remarks of others.  d. Explain their own ideas and understanding	1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 4 topics and texts, building on others' ideas and expressing their own clearly.  a. Come to discussions prepared, having read or studied required material; explicitly draw on that preparation and other information known about the topic to explore ideas under discussion.  b. Follow agreed-upon rules for discussions and carry out assigned roles.  c. Pose and respond to specific questions to clarify or follow up on information, and make comments that contribute to the discussion and link to the remarks of others.  d. Review the key ideas expressed and explain their own ideas and understanding in light of the discussion.	1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 5 topics and texts, building on others' ideas and expressing their own clearly.  a. Come to discussions prepared, having read or studied required material; explicitly draw on that preparation and other information known about the topic to explore ideas under discussion.  b. Follow agreed-upon rules for discussions and carry out assigned roles.  c. Pose and respond to specific questions by making comments that contribute to the discussion and elaborate on the remarks of others.  d. Review the key ideas expressed and draw conclusions in light of information and knowledge gained from the discussions.
<ul> <li>in light of the discussion.</li> <li>2. Determine the main ideas and supporting details of a text read aloud or information presented in diverse media and formats, including visually, quantitatively, and orally.</li> <li>3. Ask and answer questions about information from a speaker, offering appropriate elaboration or explanations and detail.</li> </ul>	<ul> <li>2. Paraphrase portions of a text read aloud or information presented in diverse media and formats, including visually, quantitatively, and orally.</li> <li>3. Identify the reasons and evidence a speaker provides to support particular points (e.g., using a graphic organizer to show connections between reasons given and support provided).</li> </ul>	<ol> <li>Summarize a written text read aloud or information presented in diverse media and formats, including visually, quantitatively, and orally.</li> <li>Summarize the points a speaker makes and explain how each claim is supported by reasons and evidence (e.g., use a graphic organizer or note cards completed while listening to summarize or paraphrase key ideas presented by a speaker).</li> </ol>

Grade 3 students:	Grade 4 students:	Grade 5 students:
Presentation of Knowledge and Ideas	Presentation of Knowledge and Ideas	Presentation of Knowledge and Ideas
<b>4.</b> Report on a topic or text, tell a story, or	4. Report on a topic or text, tell a story, or	<b>4.</b> Report on a topic or text or present an
recount an experience with appropriate facts	recount an experience in an organized	opinion, sequencing ideas logically and using
and relevant, descriptive details, speaking	manner, using appropriate facts and relevant,	appropriate facts and relevant, descriptive
clearly at an understandable pace.	descriptive details to support main ideas or	details to support main ideas or themes;
	themes; speak clearly at an understandable	speak clearly at an understandable pace.
	pace.	
5. Create audio recordings of stories or	<b>5.</b> Add audio recordings and visual displays to	5. Include multimedia components (e.g.,
poems that demonstrate fluid reading at an	presentations when appropriate to enhance	graphics, sound) and visual displays in
understandable pace; use techniques that	the development of main ideas or themes and	presentations when appropriate to enhance
engage the listener (e.g., inflection, different	to engage listeners more fully.	the development of main ideas or themes
voices); and add visual displays when		and to engage listeners more fully.
appropriate to emphasize or enhance certain		
facts or details.		
<b>6.</b> Speak in complete sentences when	<b>6.</b> Differentiate between contexts that call for	<b>6.</b> Adapt speech to a variety of contexts and
appropriate to task and situation in order to	formal English (e.g., presenting ideas) and	tasks, using formal English when appropriate
provide requested detail or clarification. (See	situations where informal discourse is	to task and situation. (See grade 5 Language
grade 3 Language standards 1 and 3 for	appropriate (e.g., small-group discussion); use	standards 1 and 3 for specific expectations.)
specific expectations.)	formal English when appropriate to task and	
	situation. (See grade 4 Language standards 1	
	and 3 for specific expectations.)	

The following standards for grades K-5 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades. Beginning in grade 3, skills and understandings that are particularly likely to require continued attention in higher grades as they are applied to increasingly sophisticated writing and speaking are marked with an asterisk (\*). For a complete list see the Literacy Progressive Skills table in the Introduction.

Kindergartners:	Grade 1 students:	Grade 2 students:
Conventions of Standard English	Conventions of Standard English	Conventions of Standard English
1. Demonstrate command of the conventions	1. Demonstrate command of the conventions	1. Demonstrate command of the conventions
of standard English grammar and usage when	of standard English grammar and usage when	of standard English grammar and usage when
writing or speaking.	writing or speaking.	writing or speaking.
a. Print many upper- and lowercase letters.	a. Print all upper- and lowercase letters.	a. Use collective nouns (e.g., group).
b. Use frequently occurring nouns and verbs.	b. Use common, proper, and possessive	b. Form and use frequently occurring irregular
c. Form regular plural nouns orally by adding	nouns.	plural nouns (e.g., feet, children, teeth, mice,
/s/ or /es/ (e.g., dog, dogs; wish, wishes).	c. Use singular and plural nouns with	fish).
d. Understand and use question words	matching verbs in basic sentences (e.g., He	c. Use reflexive pronouns (e.g., myself,
(interrogatives) (e.g., who, what, where,	hops; We hop).	ourselves).
when, why, how).	d. Use personal, possessive, and indefinite	d. Form and use the past tense of frequently
e. Use the most frequently occurring	pronouns (e.g., I me, my; they, them, their,	occurring irregular verbs (e.g., sat, hid, told).
prepositions (e.g., to, from, in, out, on, off, for,	anyone, everything).	e. Use adjectives and adverbs, and choose
of, by, with).	e. Use verbs to convey a sense of past,	between them depending on what is to be
f. Produce and expand complete sentences in	present, and future (e.g., Yesterday I walked	modified.
shared language activities.	home; Today I walk home; Tomorrow I will	f. Produce, expand, and rearrange complete
	walk home).	simple and compound sentences (e.g., The
	f. Use frequently occurring adjectives.	boy watched the movie; The little boy
	g. Use frequently occurring conjunctions (e.g.,	watched the movie; The action movie was
	and, but, or, so, because).	watched by the little boy).
	h. Use determiners (e.g., articles,	
	demonstratives).	
	i. Use frequently occurring prepositions (e.g.,	
	during, beyond, toward).	
	j. Produce and expand complete simple and	
	compound declarative, interrogative,	

Kindergartners:	Grade 1 students:	Grade 2 students:
	imperative, and exclamatory sentences in response to prompts.	
<ul> <li>2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.</li> <li>a. Capitalize the first word in a sentence, the first letter of the student's name, and the pronoun <i>I</i>.</li> <li>b. Recognize and name end punctuation.</li> <li>c. Write a letter or letters for most consonant and short-vowel sounds (phonemes).</li> <li>d. Spell simple words phonetically, drawing on knowledge of sound-letter relationships.</li> </ul>	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Capitalize dates and names of people. b. Use end punctuation for sentences. c. Use commas in dates and to separate single words in a series. d. Use conventional spelling for words with common spelling patterns and for frequently occurring irregular words. e. Spell untaught words phonetically, drawing on phonemic awareness and spelling conventions.	<ul> <li>2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.</li> <li>a. Capitalize holidays, product names, and geographic names.</li> <li>b. Use commas in greetings and closing of letters.</li> <li>c. Use an apostrophe to form contractions and frequently occurring possessives.</li> <li>d. Generalize learned spelling patterns when writing words (e.g., cage → badge; boy → boil).</li> <li>e. Consult reference materials, including beginning dictionaries, as needed to check and correct spellings.</li> </ul>
Knowledge of Language	Knowledge of Language	Knowledge of Language
3. (Begins in grade 2)	<b>3.</b> (Begins in grade 2)	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Compare formal and informal uses of English.
Vocabulary Acquisition and Use	Vocabulary Acquisition and Use	Vocabulary Acquisition and Use
4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on kindergarten reading and content.  a. Identify new meanings for familiar words and apply them accurately (e.g., knowing duck is a bird and learning the verb to duck).  b. Use the most frequently occurring inflections and affixes (e.g., -ed, -s, re-, un-,	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grade 1 reading and content, choosing flexibly from an array of strategies.  a. Use sentence-level context as a clue to the meaning of a word or phrase.  b. Use frequently occurring affixes as a clue to the meaning of a word.	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grade 2 reading and content, choosing flexibly from an array of strategies.  a. Use sentence-level context as a clue to the meaning of a word or phrase.

Kindergartners:	Grade 1 students:	Grade 2 students:
pre-, -ful, -less) as a clue to the meaning of an unknown word.	c. Identify frequently occurring root words (e.g., look) and their inflectional forms (e.g., looks, looked, looking).	b. Determine the meaning of the new word formed when a known prefix is added to a known word (e.g., happy/unhappy, tell/retell). c. Use a known root word as a clue to the meaning of an unknown word with the same root (e.g., addition, additional). d. Use knowledge of the meaning of individual words to predict the meaning of compound words (e.g., birdhouse, lighthouse, housefly; bookshelf, notebook, bookmark). e. Use glossaries and beginning picture dictionaries, both print and digital, to determine or clarify the meaning of words and phrases.
5. With guidance and support from adults, explore word relationships and nuances in word meanings.  a. Sort common objects into categories (e.g., shapes, foods) to gain a sense of the concepts the categories represent.  b. Demonstrate understanding of frequently occurring verbs and adjectives by relating them to their opposites (antonyms).  c. Identify real-life connections between words and their use (e.g., note places at school that are colorful).  d. Distinguish shades of meaning among verbs describing the same general action (e.g., walk, march, strut, prance) by acting out the meanings.	5. With guidance and support from adults, demonstrate understanding of word relationships and nuances in word meanings. a. Sort words into categories (e.g., colors, clothing) to gain a sense of the concepts the categories represent. b. Define words by category and by one or more key attributes (e.g., a duck is a bird that swims; a tiger is a large cat with stripes). c. Identify real-life connections between words and their use (e.g., note places at home that are cozy). d. Distinguish shades of meaning among verbs differing in manner (e.g., look, peek, glance, stare, glare, scowl) and adjectives differing in intensity (e.g., large, gigantic) by defining or choosing them or by acting out the meanings.	5. Demonstrate understanding of word relationships and nuances in word meanings. a. Identify real-life connections between words and their use (e.g., describe foods that are spicy or juicy). b. Distinguish shades of meaning among closely related verbs (e.g., toss, throw, hurl) and closely related adjectives (e.g., thin, slender, skinny, scrawny).
<b>6.</b> Use words and phrases acquired through conversations, reading and being read to, and responding to texts.	<b>6.</b> Use words and phrases acquired through conversations, reading and being read to, and responding to texts, including using frequently	<b>6.</b> Use words and phrases acquired through conversations, reading and being read to, and responding to texts, including using adjectives

Kindergartners:	Grade 1 students:	Grade 2 students:
		and adverbs to describe (e.g., When other kids are happy that makes me happy).

Grade 3 students:	Grade 4 students:	Grade 5 students:
Conventions of Standard English	Conventions of Standard English	Conventions of Standard English
<b>1.</b> Demonstrate command of the conventions	1. Demonstrate command of the conventions	1. Demonstrate command of the conventions
of standard English grammar and usage when	of standard English grammar and usage	of standard English grammar and usage
writing or speaking.	when writing or speaking.	when writing or speaking.
a. Use nouns, pronouns, verbs, adjectives,	a. Use nouns, pronouns, verbs, adjectives,	a. Use nouns, pronouns, verbs, adjectives,
and adverbs appropriate to function and	adverbs, relative pronouns (who, whose,	adverbs, relative pronouns, relative adverbs,
purpose in order to apply the conventions of	whom, which, that), and relative adverbs	conjunctions, prepositions, and interjections
English.	(where, when, why) appropriate to function	appropriate to function and purpose in order
b. Form and use regular and irregular plural	and purpose in order to apply the	to apply the conventions of English.
nouns.	conventions of English.	b. Form and use the perfect (e.g., I had
c. Use abstract nouns (e.g., childhood).	b. Form and use the progressive (e.g., I was	walked; I have walked; I will have walked)
d. Form and use regular and irregular verbs.	walking; I am walking; I will be walking) verb	verb tenses.
e. Form and use the simple (e.g., I walked; I	tenses.	c. Use verb tense to convey various times,
walk; I will walk) verb tenses.	c. Use modal auxiliaries (e.g., can, may, must)	sequences, states, and conditions.
f. Ensure subject-verb and pronoun-	to convey various conditions.	d. Recognize and correct inappropriate shifts
antecedent agreement.*	d. Order adjectives within sentences	in verb tense.*
g. Form and use comparative and superlative	according to conventional patterns (e.g., a	e. Use correlative conjunctions (e.g., either/or,
adjectives and adverbs, and choose between	small red bag rather than a red small bag).	neither/nor).
them depending on what is to be modified.	e. Form and use prepositional phrases.	
h. Use coordinating and subordinating	f. Produce complete sentences, recognizing	
conjunctions.	and correcting inappropriate fragments and	
i. Produce simple, compound, and complex	run-ons.*	
sentences.	g. Correctly use frequently confused words	
	(e.g., to, too, two, there, their).*	

Grade 3 students:	Grade 4 students:	Grade 5 students:
<b>2.</b> Demonstrate command of the conventions	2. Demonstrate command of the conventions	2. Demonstrate command of the conventions
of standard English capitalization,	of standard English capitalization,	of standard English capitalization, punctuation,
punctuation, and spelling when writing.	punctuation, and spelling when writing.	and spelling when writing.
a. Capitalize appropriate words in titles.	a. Use correct capitalization.	a. Use punctuation to separate items in a
b. Use commas in addresses.	b. Use commas and quotation marks to mark	series.*
c. Use commas and quotation marks in	direct speech and quotations from a text.	b. Use a comma to separate an introductory
dialogue.	c. Use a comma before a coordinating	element from the rest of the sentence.
d. Form and use possessives.	conjunction in a compound sentence.	c. Use a comma to set off the words yes and
e. Use conventional spelling for high-	d. Spell grade-appropriate words correctly,	no (e.g., Yes, thank you), to set off a tag
frequency and other studied words and for	consulting references as needed.	question from the rest of the sentence (e.g.,
adding suffixes to base words (e.g., sitting,		It's true, isn't it?), and to indicate direct
smiled, cries, happiness).		address (e.g., Is that you, Steve?).
f. Use spelling patterns and generalizations		d. Use underlining, quotation marks, or italics
(e.g., word families, position-based spellings,		to indicate titles of works.
syllable patterns, ending rules, meaningful		e. Spell grade-appropriate words correctly,
word parts) in writing words.		consulting references as needed.
g. Consult reference materials, including		
beginning dictionaries, as needed to check		
and correct spelling.		
Knowledge of Language	Knowledge of Language	Knowledge of Language
3. Use knowledge of language and its	3. Use knowledge of language and its	3. Use knowledge of language and its
conventions when writing, speaking, reading,	conventions when writing, speaking, reading,	conventions when writing, speaking, reading,
or listening.	or listening.	or listening.
a. Choose words and phrases for effect.*	a. Choose words and phrases to convey ideas	a. Expand, combine, and reduce sentences for
b. Recognize and observe differences	precisely.*	meaning, reader/listener interest, and style.
between the conventions of spoken and	b. Choose punctuation for effect.*	b. Compare and contrast the varieties of
written standard English.	c. Differentiate between contexts that call for	English (e.g., dialects, registers) used in stories,
	formal English (e.g., presenting ideas) and	dramas, or poems.
	situations where informal discourse is	
	appropriate (e.g., small-group discussion).	

Grade 3 students:	Grade 4 students:	Grade 5 students:
Vocabulary Acquisition and Use	Vocabulary Acquisition and Use	Vocabulary Acquisition and Use
4. Determine or clarify the meaning of unknown and multiple-meaning word and phrases based on grade 3 reading and content, choosing flexibly from a range of strategies.  a. Use a sentence-level context as a clue to the meaning of a word or phrase.  b. Determine the meaning of a new word formed when a known affix is added to a known word (e.g., agreeable/disagreeable, comfortable/uncomfortable, care/careless, heat/preheat).  c. Use a known root word as a clue to the meaning of an unknown word with the same root (e.g., company, companion).  d. Use glossaries or beginning dictionaries, both print and digital, to determine or clarify the precise meaning of key words and phrases.	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grade 4 reading and content, choosing flexibly from a range of strategies.  a. Determine meaning of unfamiliar words by using knowledge of phonetics, word structure, and language structure through reading words in text (word order, grammar, syntax), use context (e.g., definitions, examples, or restatements in text) as a clue to the meaning of a word or phrase.  b. Use common, grade-appropriate Greek and Latin affixes and roots as clues to the meaning of a word (e.g., telegraph, photograph, autograph).  c. Consult reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation and determine or clarify the precise meaning of key words and phrases.	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grade 5 reading and content, choosing flexibly from a range of strategies.  a. Determine meaning of unfamiliar words by using knowledge of word structure (root words, prefixes, suffixes, abbreviations) and language structure through reading words in text (word order, grammar, syntax), use context (e.g., cause/effect relationships and comparisons in text) as a clue to the meaning of a word or phrase.  b. Use common, grade-appropriate Greek and Latin affixes and roots as clues to the meaning of a word (e.g., photograph, photosynthesis).  c. Consult reference materials (e.g., dictionaries, glossaries, thesauruses), both printing and digital, to find the pronunciation and determine or clarify the precise meaning of key words and phrases.

Grade 3 students:	Grade 4 students:	Grade 5 students:
<b>5.</b> Demonstrate understanding of word relationships and nuances in word meanings. a. Distinguish the literal and nonliteral meanings of words and phrases in context (e.g., take steps). b. Identify real-life connections between words and their use (e.g., describe people who are <i>friendly</i> or <i>helpful</i> ). c. Distinguish shades of meaning among related words (e.g., words that describe states of mind or degrees of certainty, such as <i>knew</i> , <i>believed</i> ).	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings.  a. Explain the meaning of simple similes and metaphors (e.g., as pretty as a picture) in context.  b. Recognize and explain the meaning of common idioms, adages, and proverbs.  c. Demonstrate understanding of words by relating them to their opposites (antonyms) and to words with similar but not identical meanings (synonyms).	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figurative language, including similes and metaphors, in context. b. Recognize and explain the meaning of common idioms, adages, and proverbs. c. Use the relationship between particular words (e.g., synonyms, antonyms, homographs) to better understand each of the words.
<b>6.</b> Acquire and accurately use gradeappropriate conversational, general academic, and domain-specific words and phrases, including those that signal spatial and temporal relationships (e.g., After dinner that night we went looking for them).	6. Acquire and accurately use grade- appropriate general academic and domain- specific words and phrases, including those that signal precise actions, emotions, or states of being (e.g., quizzed, whined, stammered) and that are basic to a particular topic (e.g., wildlife, conservation, and endangered when discussing animal preservation).	<b>6.</b> Acquire and accurately use gradeappropriate general academic and domainspecific words and phrases, including those that signal contrast, addition, and other logical relationships (e.g., however, although, nevertheless, similarly, moreover, in addition).

# **6-12 English/Language Arts Standards**

## **Reading Standards for Literature 6-12**

The following standards offer a focus for instruction each year and help ensure that students gain adequate exposure to a range of texts and tasks. Rigor is also infused through the requirement that students read increasingly complex texts through the grades. Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.

Grade 6 students:	Grade 7 students:	Grade 8 students:
Key Ideas and Details	Key Ideas and Details	Key Ideas and Details
1. Cite textual evidence to support analysis of	Cite several pieces of textual evidence to	Cite the textual evidence that most
what the text says explicitly as well as	support analysis of what the text says	strongly supports an analysis of what the text
inferences drawn from the text.	explicitly as well as inferences drawn from	says explicitly as well as inferences drawn
	the text.	from the text.
2. Determine a theme or central idea of a text	2. Determine a theme or central idea of a text	2. Determine a theme or central idea of a
and how it is conveyed through particular	and analyze its development over the course	text and analyze its development over the
details; restate and summarize main ideas or	of the text; restate and summarize main ideas	course of the text, including its relationship
events, in correct sequence, after reading a	or events, in correct sequence, after reading	to the characters, setting, and plot; restate
text.	a text.	and summarize main ideas or events, in
		correct sequence, after reading a text.
<b>3.</b> Describe how a particular story's or	<b>3.</b> Analyze how particular elements of a story	<b>3.</b> Analyze how particular lines of dialogue or
drama's plot unfolds in a series of episodes as	or drama interact (e.g., how setting shapes	incidents in a story or drama propel the
well as how the characters respond or change	the characters or plot).	action, reveal aspects of a character, or
as the plot moves toward a resolution or as		provoke a decision.
the narrative advances.		
Craft and Structure	Craft and Structure	Craft and Structure
4. Determine the meaning of words and	<b>4.</b> Determine the meaning of words and	<b>4.</b> Determine the meaning of words and
phrases as they are used in a text, including	phrases as they are used in a text, including	phrases as they are used in a text, including
figurative and connotative meanings. Analyze	figurative and connotative meanings; analyze	figurative and connotative meanings; analyze
the impact of a specific word choice on	the impact of a specific word choice on	the impact of specific word choices on
meaning and tone.	meaning and tone.	meaning and tone, including analogies or
		allusions to other texts.
<b>5.</b> Analyze the overall structure of a text:	<b>5.</b> Analyze the overall structure of a text: how	<b>5.</b> Analyze the overall structure of a text:
how a particular sentence, chapter, scene,	a drama's or poem's form or structure (e.g.	compare and contrast the structure of two or
or stanza fits into the overall work and	sonnet, soliloquy) contributes to its meaning,	more texts and analyze how the differing
analyzing how it contributes to the	including analyzing the impact of rhymes and	structure of each text contributes to its
		meaning and style.

Grade 6 students:	Grade 7 students:	Grade 8 students:
development of the characters, theme,	other repetitions of sounds (e.g., alliteration)	
setting, or plot.	on a specific verse or stanza of a poem or	
	section of a story or drama.	
<b>6.</b> Determine author's purpose and explain	<b>6.</b> Analyze author's purpose and how an	<b>6.</b> Analyze author's purpose and how
how an author develops the point of view of	author establishes and contrasts the points of	differences in the points of view of the
the narrator or speaker in a text.	view of different characters or narrators in a	characters and the audience or reader (e.g.,
	text.	created through the use of dramatic irony)
		create such effects as suspense or humor.
Integration of Knowledge and Ideas	Integration of Knowledge and Ideas	Integration of Knowledge and Ideas
7. Compare and contrast the experience of	7. Compare and contrast a written story,	7. Analyze the extent to which a filmed or live
reading a story, drama, or poem to listening	drama, or poem to its audio, filmed, staged,	production of a story or drama stays faithful
to or viewing an audio, video, or live version	or multimedia version, analyzing the effects	to or departs from the text or script,
of the text, including contrasting what they	of techniques unique to each medium (e.g.,	evaluating the choices made by the director
"see" and "hear" when reading the text to	lighting, sound, color, or camera focus and	or actors (e.g., Old Yeller, Brian's Song, The
what they perceive when they listen or watch	angles in a film) (e.g., The Incredible Journey,	Miracle Worker).
(e.g., Shiloh).	Call of the Wild).	
8. (Not applicable to literature)	8. (Not applicable to literature)	8. (Not applicable to literature)
9. Compare and contrast texts in different	9. Compare and contrast a fictional portrayal	<b>9.</b> Analyze how a modern work of fiction
forms or genres (e.g., stories and poems;	of a time, place, or character and a historical	draws on themes, patterns of events, or
historical novels and fantasy stories) in terms	or cultural account of the same period as a	character types from myths, traditional
of their approaches to similar themes and	means of understanding how authors of	stories, or religious works such as the Bible,
topics.	fiction use or alter history.	including describing how the material is
		rendered new.
Range of Reading and Level of Text	Range of Reading and Level of Text	Range of Reading and Level of Text
Complexity	Complexity	Complexity
<b>10.</b> By the end of the year, read and	<b>10.</b> By the end of the year, read and	<b>10.</b> By the end of the year, read and
comprehend a range of literature from a	comprehend a range of literature from a	comprehend a range of literature from a
variety of cultures, within a complexity band	variety of cultures, within a complexity band	variety of cultures, within a complexity band
appropriate to grade 6 (from upper grade 5 to	appropriate to grade 7 (from upper grade 6 to	appropriate to grade 8 (from upper grade 7 to
grade 7), with scaffolding as needed at the	grade 8), with scaffolding as needed at the	grade 9), with scaffolding as needed at the
high end of the range.	high end of the range.	high end of the range.

### **Reading Standards for Literature 6-12**

The anchor standards and high school grade-specific standards work in tandem to define expectations—the former providing broad standards, the latter providing additional specificity.

Grades 9-10 students:	Grades 11-12 students:
Key Ideas and Details	Key Ideas and Details
1. Cite strong and thorough textual evidence to support analysis of	1. Cite strong and thorough textual evidence to support analysis of
what the text says explicitly as well as implicit inferences drawn from	what the text says explicitly as well as inferences drawn from the text,
the text.	including determining where the text leaves matters uncertain
	(ambiguity).
2. Determine a theme or central idea of a text and analyze in detail its	2. Determine two or more themes or central ideas of a text and
development over the course of the text, including how it emerges	analyze their development over the course of the text, including how
and is shaped and refined by specific details; restate and summarize	they interact and build on one another to produce a complex
main ideas or events, in correct sequence, after reading a text.	account; restate and summarize main ideas or events, in correct
	sequence, after reading a text.
<b>3.</b> Analyze how complex characters (e.g., those with multiple or	<b>3.</b> Analyze the impact of the author's choices regarding how to
conflicting motivations) develop over the course of a text, interact	develop and relate elements of a story or drama (e.g., where a story is
with other characters, and advance the plot or develop the theme.	set, how the action is ordered, how the characters are introduced and
	developed).
Craft and Structure	Craft and Structure
<b>4.</b> Determine the meaning of words and phrases as they are used in	<b>4.</b> Determine the meaning of words and phrases as they are used in
the text, including figurative and connotative meanings; analyze the	the text, including figurative and connotative meanings; analyze the
cumulative impact of specific word choices on meaning and tone	impact of specific word choices on meaning and tone, including words
(e.g., how the language evokes a sense of time and place; how it sets	with multiple meanings or language that is particularly fresh, engaging,
a formal or informal tone).	or beautiful. (Include Shakespeare as well as other authors.)
<b>5.</b> Analyze how an author's choices concerning how to structure a	<b>5.</b> Analyze how an author's choices concerning how to structure
text, use literary devices appropriate to genre (e.g., foreshadowing,	specific parts of a text (e.g., the choice of where to begin or end a
imagery, allusion or symbolism), order events within it (e.g., parallel	story, the choice to provide a comedic or tragic resolution) contribute
plots), and manipulate time (e.g., pacing, flashbacks) create such	to its overall structure and meaning as well as its aesthetic impact.
effects as mystery, tension, or surprise.	
<b>6</b> . Analyze a particular point of view or cultural experience reflected	<b>6.</b> Analyze a case in which grasping point of view requires
in a work of literature (e.g., mythology, colonialism, local culture),	distinguishing what is directly stated in a text from what is really
drawing on a wide reading of world literature.	meant (e.g., satire, sarcasm, irony, understatement, or attitude).
Integration of Knowledge and Ideas	Integration of Knowledge and Ideas

Grades 9-10 students:	Grades 11-12 students:
7. Analyze the representation of a subject or a key scene in two different artistic media, including what is emphasized or absent in each treatment (e.g., Auden's "Musée des Beaux Arts" and Breughel's Landscape with the Fall of Icarus).	7. Analyze multiple interpretations of a story, drama, or poem (e.g., recorded or live production of a play or recorded novel or poetry), evaluating how each version interprets the source text. (Include at least one play by Shakespeare and one play by an American dramatist.)
8. (Not applicable to literature)	8. (Not applicable to literature)
<b>9.</b> Analyze how an author draws on and transforms source material in a specific work (e.g., how Shakespeare treats a theme or topic from Ovid or how a later author draws on a play by Shakespeare).	<b>9.</b> Demonstrate knowledge of eighteenth-, nineteenth- and early-twentieth-century foundational works of American literature, including how two or more texts from the same period treat similar themes or topics.
Range of Reading and Level of Text Complexity	Range of Reading and Level of Text Complexity
<b>10.</b> By the end of grade 9, read and comprehend a range of literature from a variety of cultures, within a complexity band appropriate to grade 9 (from upper grade 8 to grade 10), with scaffolding as needed at the high end of the range.	<b>10.</b> By the end of grade 11, read and comprehend a range of literature from a variety of cultures, within a complexity band appropriate to grade 11 (from upper grade 10 to grade 12), with scaffolding as needed at the high end of the range.
By the end of grade 10, read and comprehend a range of literature from a variety of cultures, within a complexity band appropriate to grade 10 (from upper grade 9 to grade 11), with scaffolding as needed at the high end of the range.	By the end of grade 12, read and comprehend a range of literature from a variety of cultures, at the high end of the grades 11–12 text complexity band independently and proficiently.

## **Reading Standards for Informational Text 6-12**

Grade 6 students:	Grade 7 students:	Grade 8 students:
Key Ideas and Details	Key Ideas and Details	Key Ideas and Details
<ol> <li>Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.</li> <li>Determine a central idea and subtopics of a text and how they are conveyed through particular details; restate and summarize the</li> </ol>	<ol> <li>Cite several pieces of textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.</li> <li>Determine the central idea and subtopics in a text and analyze their development over the course of the text; restate and summarize</li> </ol>	<ol> <li>Cite the textual evidence that most strongly supports an analysis of what the text says explicitly as well as inferences drawn from the text.</li> <li>Determine a central idea and subtopics of a text and analyze their development over the course of the text, including their relationship</li> </ol>
central idea or events, in correct sequence when necessary, after reading a text.	the central idea or events, in correct sequence when necessary, after reading a text.	to supporting ideas; restate and summarize the central idea or events, in correct sequence when necessary, after reading a text.
<b>3.</b> Analyze in detail how a key individual, event, or idea is introduced, illustrated, and elaborated on in a text (e.g., through examples or anecdotes).	<b>3.</b> Analyze the interactions between individuals, events, and ideas presented in a text (e.g., how ideas influence individuals or events, or how individuals influence ideas or events).	<b>3.</b> Analyze how a text makes connections among and distinctions between individuals, ideas, or events (e.g., through comparisons, analogies, or categories).
Craft and Structure	Craft and Structure	Craft and Structure
<b>4.</b> Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings.	<b>4.</b> Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze the impact of a specific word choice on meaning and tone.	4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze the impact of specific word choices on meaning and tone, including analogies or allusions to other texts.
<ul> <li>5. Analyze how a particular sentence, paragraph, chapter, or section fits into the overall structure of a text and contributes to the development of the ideas.</li> <li>6. Determine an author's purpose (to inform, persuade, entertain, critique, etc.) and point</li> </ul>	<ul> <li>5. Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to the development of the ideas.</li> <li>6. Determine an author's purpose (to inform, persuade, entertain, critique, etc.) and point of view in a text and analyze how the author</li> </ul>	<ul> <li>5. Analyze in detail the structure of a specific paragraph in a text, including the role of particular sentences in developing and refining a key concept.</li> <li>6. Determine an author's purpose (to inform, persuade, entertain, critique, etc.) and point of view in a text and analyze how the author</li> </ul>

Grade 6 students:	Grade 7 students:	Grade 8 students:
of view in a text and explain how it is conveyed in the text.	distinguishes his or her point of view from that of others.	acknowledges and responds to conflicting evidence or viewpoints.
Integration of Knowledge and Ideas	Integration of Knowledge and Ideas	Integration of Knowledge and Ideas
7. Integrate information presented in different media (e.g., may include, but not limited to podcasts) or formats (e.g., visually, quantitatively/ data-related) as well as in words to develop a coherent understanding of a topic or issue.	7. Compare and contrast a text to an audio, video, or multimedia version of the text, analyzing each medium's portrayal of the subject (e.g., how the method of the delivery of a speech affects the impact of the overall message).	7. Evaluate the advantages and disadvantages of using different media (e.g., print or digital text, video, multimedia) to present a particular topic or idea.
8. Trace and evaluate the argument and specific claims in a text, distinguishing claims that are supported by reasons and evidence from claims that are not.	8. Trace and evaluate the argument and specific claims in a text, assessing whether the reasoning is sound and the evidence is relevant and sufficient to support the claims.	8. Delineate and evaluate the argument and specific claims in a text (e.g., identify bias and propaganda techniques, well-supported logical arguments), assessing whether the reasoning is sound and the evidence is relevant and sufficient; recognize when irrelevant evidence is introduced.
<b>9.</b> Compare and contrast one author's presentation of events with that of another (e.g., a memoir written by and a biography on the same person).	<b>9.</b> Analyze how two or more authors writing about the same topic shape their presentations of key information by emphasizing different evidence or advancing different interpretations of facts.	<b>9.</b> Analyze a case in which two or more texts provide conflicting information on the same topic and identify where the texts disagree on matters of fact or interpretation.
Range of Reading and Level of Text Complexity	Range of Reading and Level of Text Complexity	Range of Reading and Level of Text Complexity
10. By the end of the year, read and comprehend literary nonfiction, within a complexity band appropriate to grade 6 (from upper grade 5 to grade 7), with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend literary nonfiction, within a complexity band appropriate to grade 7 (from upper grade 6 to grade 8), with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend literary nonfiction, within a complexity band appropriate to grade 8 (from upper grade 7 to grade 9), with scaffolding as needed at the high end of the range.

### **Reading Standards for Informational Text 6-12**

The anchor standards and high school grade-specific standards work in tandem to define expectations—the former providing broad standards, the latter providing additional specificity.

Grades 9-10 students:	Grades 11-12 students:
	Grades II II stadents.
Key Ideas and Details	Key Ideas and Details
1. Cite strong and thorough textual evidence to support analysis of	1. Cite strong and thorough textual evidence to support analysis of
what the text says explicitly as well as inferences drawn from the text.	what the text says explicitly as well as inferences drawn from the text,
	including determining where the text leaves matters uncertain.
2. Determine a central idea of a text and analyze its development	2. Determine two or more central ideas of a text and analyze their
over the course of the text, including how it emerges and is shaped	development over the course of the text, including how they interact
and refined by specific details; restate and summarize main ideas or	and build on one another to provide a complex analysis; restate and
events, in correct sequence when necessary, after reading a text.	summarize main ideas or events, in correct sequence when
	necessary, after reading a text.
<b>3.</b> Analyze how the author unfolds an analysis or series of ideas or	<b>3.</b> Analyze a complex set of ideas or sequence of events and explain
events, including the order in which the points are made, how they	how specific individuals, ideas, or events interact and develop over
are introduced and developed, and the connections that are drawn	the course of the text.
between them.	
Craft and Structure	Craft and Structure
<b>4.</b> Determine the meaning of words and phrases as they are used in	<b>4.</b> Determine the meaning of words and phrases as they are used in
various genres, including figurative, connotative, and technical	various genres, including figurative, connotative, and technical
meanings; analyze the cumulative impact of specific word choices on	meanings; analyze how an author uses and refines the meaning of a
meaning and tone (e.g., how the language of a court opinion differs	key term or terms over the course of a text.
from that of a newspaper).	
5. Analyze in detail how an author's ideas or claims are developed	<b>5.</b> Analyze and evaluate the effectiveness of the structure an author
and refined by particular sentences, paragraphs, or larger portions of	uses in his or her exposition or argument, including whether the
a text (e.g., a section or chapter).	structure makes points clear, convincing, and engaging.
<b>6.</b> Determine an author's point of view or purpose in a text and	<b>6.</b> Discern an author's point of view or purpose in a text in which the
analyze how an author uses rhetoric to advance that point of view or	rhetoric is particularly effective, analyzing how style and content
purpose.	contribute to the power, persuasiveness, or aesthetic impact of the
	text.
Integration of Knowledge and Ideas	Integration of Knowledge and Ideas
7. Analyze various accounts of a subject told in different media (e.g., a	7. Integrate and evaluate multiple sources of information presented
person's life story in both print and multimedia), determining which	in different media or formats (e.g., visually, quantitatively) as well as
details are emphasized in each account.	in words in order to address a question or solve a problem.

## Reading Standards for Informational Text 6-12

Grades 9-10 students:	Grades 11-12 students:
8. Delineate and evaluate the argument and specific claims in a text (e.g., bias and propaganda techniques, emotional effect of specific word choices and sentence structures, well-supported logical arguments), assessing whether the reasoning is valid and the evidence is relevant and sufficient; identify false statements and	<b>8.</b> Delineate and evaluate the reasoning in seminal U.S. texts, including the application of constitutional principles and use of legal reasoning (e.g., in U.S. Supreme Court majority opinions and dissents) and the premises, purposes, and arguments in works of public advocacy (e.g., <i>The Federalist</i> , presidential addresses).
fallacious reasoning.  9. Analyze seminal U.S. and world documents of historical and literary significance (e.g., Washington's Farewell Address, the Gettysburg Address, Roosevelt's Four Freedoms speech, King's "Letter from Birmingham Jail"), including how they address related themes and concepts.	<b>9.</b> Analyze seventeenth-, eighteenth-, and nineteenth-century foundational U.S. and world documents of historical and literary significance (including The Declaration of Independence, the Preamble to the Constitution, the Bill of Rights, and Lincoln's Second Inaugural Address) for their themes, purposes, and rhetorical features.
Range of Reading and Level of Text Complexity	Range of Reading and Level of Text Complexity
<b>10.</b> By the end of grade 9, read and comprehend literary nonfiction, within a complexity band appropriate to grade 9 (from upper grade 8 to grade 10), with scaffolding as needed at the high end of the range.	<b>10.</b> By the end of grade 11, read and comprehend literary nonfiction, within a complexity band appropriate to grade 11 (from upper grade 10 to grade 12), with scaffolding as needed at the high end of the range.
By the end of grade 10, read and comprehend literary nonfiction, within a complexity band appropriate to grade 10 (from upper grade 9 to grade 11), with scaffolding as needed at the high end of the range.	By the end of grade 12, read and comprehend literary nonfiction at the high end of the grades 11–12 text complexity band independently and proficiently.

## **Writing Standards 6-12**

The following standards for 6-12 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. Each year in their writing, students should demonstrate increasing sophistication in all aspects of language use, from vocabulary and syntax to the development and organization of ideas, and they should address increasingly demanding content and sources. Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.

Grade 6 students:	Grade 7 students:	Grade 8 students:
Text Types and Purposes	Text Types and Purposes	Text Types and Purposes
1. Write arguments to support claims with	1. Write arguments to support claims with	1. Write arguments to support claims with
clear reasons and relevant evidence.	clear reasons and relevant evidence.	clear reasons and relevant evidence.
a. Introduce claim(s) and organize the	a. Introduce claim(s), acknowledge alternate	a. Introduce claim(s), acknowledge and
reasons and evidence clearly.	or opposing claims, and organize the reasons	distinguish the claim(s) from alternate or
b. Support claim(s) with clear reasons and	and evidence logically.	opposing claims, and organize the reasons
accurate, relevant evidence, using credible	b. Support claim(s) with logical reasoning and	and evidence logically.
sources and demonstrating an understanding	accurate, relevant evidence, using credible	b. Support claim(s) with logical reasoning and
of the topic or text.	sources and demonstrating an understanding	accurate, relevant evidence, using credible
c. Use words, phrases, and clauses to clarify	of the topic or text.	sources and demonstrating an understanding
the relationships among claim(s) and reasons.	c. Use words, phrases, and clauses to create	of the topic or text.
d. Establish and maintain a formal style.	cohesion and clarify the relationships among	c. Use words, phrases, and clauses to create
e. Provide a concluding statement or section	claim(s), reasons, and evidence.	cohesion and clarify the relationships among
that follows from the argument presented.	d. Establish and maintain a formal style.	claim(s), counterclaims, reasons, and
	e. Provide a concluding statement or section	evidence.
	that follows from and supports the argument	d. Establish and maintain a formal style.
	presented.	e. Provide a concluding statement or section
		that follows from and supports the argument
		presented.
2. Write informative/explanatory texts to	2. Write informative/explanatory texts to	2. Write informative/explanatory texts to
examine a topic and convey ideas, concepts,	examine a topic and convey ideas, concepts,	examine a topic and convey ideas, concepts,
and information through the selection,	and information through the selection,	and information through the selection,
organization, and analysis of relevant	organization, and analysis of relevant	organization, and analysis of relevant
content.	content.	content.
a. Introduce a topic; organize ideas, concepts,	a. Introduce a topic clearly, previewing what	a. Introduce a topic clearly, previewing what
and information, using strategies such as	is to follow; organize ideas, concepts, and	is to follow; organize ideas, concepts, and
definition, classification,	information, using strategies such as	information into broader categories; include

Grade 6 students:	Grade 7 students:	Grade 8 students:
comparison/contrast, and cause/effect; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension.	definition, classification, comparison/contrast, and cause/effect; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension.	formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension.
b. Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples. c. Use appropriate transitions to clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Establish and maintain a formal style. f. Provide a concluding statement or section that follows from the information or explanation presented.	b. Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples. c. Use appropriate transitions to create cohesion and clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Establish and maintain a formal style. f. Provide a concluding statement or section that follows from and supports the information or explanation presented.	b. Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples. c. Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Establish and maintain a formal style. f. Provide a concluding statement or section that follows from and supports the information or explanation presented.

Grade 6 students:	Grade 7 students:	Grade 8 students:
3. Use narrative writing to develop real or imagined experiences or events using effective technique, relevant descriptive details, and well-structured event sequences.  a. Engage and orient the reader by establishing a context and introducing a narrator and/or characters; organize an event sequence that unfolds naturally and logically.  b. Use narrative techniques, such as dialogue, pacing, and description, to develop experiences, events, and/or characters.  c. Use a variety of transition words, phrases, and clauses to convey sequence and signal shifts from one time frame or setting to another.  d. Use precise words and phrases, relevant descriptive details, and sensory language to convey experiences and events.  e. Provide a conclusion that follows from the narrated experiences or events.	3. Use narrative writing to develop real or imagined experiences or events using effective technique, relevant descriptive details, and well-structured event sequences.  a. Engage and orient the reader by establishing a context and point of view and introducing a narrator and/or characters; organize an event sequence that unfolds naturally and logically.  b. Use narrative techniques, such as dialogue, pacing, and description, to develop experiences, events, and/or characters.  c. Use a variety of transition words, phrases, and clauses to convey sequence and signal shifts from one time frame or setting to another.  d. Use precise words and phrases, relevant descriptive details, and sensory language to advance the action and convey experiences and events.  e. Provide a conclusion that follows from and reflects on the narrated experiences or events.	3. Use narrative writing to develop real or imagined experiences or events using effective technique, relevant descriptive details, and well-structured event sequences.  a. Engage and orient the reader by establishing a context and point of view and introducing a narrator and/or characters; organize an event sequence that unfolds naturally and logically.  b. Use narrative techniques, such as dialogue, pacing, description, and reflection, to develop experiences, events, and/or characters.  c. Use a variety of transition words, phrases, and clauses to convey sequence, signal shifts from one time frame or setting to another, and show the relationships among experiences and events.  d. Use precise words and phrases, relevant descriptive details, and sensory language to advance the action and convey experiences and events.  e. Provide a conclusion that follows from and reflects on the narrated experiences or
		events.

### Writing Standards 6-12

Grade 6 students:	Grade 7 students:	Grade 8 students:
Production and Distribution of Writing	Production and Distribution of Writing	Production and Distribution of Writing
4. Produce clear and coherent writing in	4. Produce clear and coherent writing in	4. Produce clear and coherent writing in
which the development, organization, and	which the development, organization, and	which the development, organization, and
style are appropriate to task, purpose, and	style are appropriate to task, purpose, and	style are appropriate to task, purpose, and
audience. (Grade-specific expectations for	audience. (Grade-specific expectations for	audience. (Grade-specific expectations for
writing types are defined in standards 1–3	writing types are defined in standards 1–3	writing types are defined in standards 1–3
above.)	above.)	above.)
5. With some guidance and support from	5. With some guidance and support from	5. With some guidance and support from
peers and adults, develop and strengthen	peers and adults, develop and strengthen	peers and adults, develop and strengthen
writing as needed by planning, revising,	writing as needed by planning, revising,	writing as needed by planning, revising,
editing, rewriting, or trying a new approach.	editing, rewriting, or trying a new approach,	editing, rewriting, or trying a new approach,
(Editing for conventions should demonstrate	focusing on how well purpose and audience	focusing on how well purpose and audience
command of Language standards 1–3 up to	have been addressed. (Editing for	have been addressed. (Editing for
and including grade 6.)	conventions should demonstrate command	conventions should demonstrate command
	of Language standards 1–3 up to and	of Language standards 1–3 up to and
	including grade 7.)	including grade 8.)
<b>6.</b> Use technology, including the Internet, to	<b>6.</b> Use technology, including the Internet, to	<b>6.</b> Use technology, including the Internet, to
produce and publish writing as well as to	produce and publish writing and link to and	produce and publish writing and present the
interact and collaborate with others;	cite sources as well as to interact and	relationships between information and ideas
demonstrate sufficient command of	collaborate with others, including linking to	efficiently as well as to interact and
keyboarding skills to create a piece of writing.	and citing sources.	collaborate with others.
Research to Build and Present Knowledge	Research to Build and Present Knowledge	Research to Build and Present Knowledge
7. Conduct short research projects to answer	7. Conduct short research projects to answer	7. Conduct short research projects to answer
a question, drawing on several sources and	a question, drawing on several sources and	a question (including a self-generated
refocusing the inquiry when appropriate.	generating additional related, focused	question), drawing on several sources and
	questions for further research and	generating additional related, focused
	investigation.	questions that allow for multiple avenues of
		exploration.
<b>8.</b> Gather relevant information from multiple	8. Gather relevant information from multiple	8. Gather relevant information from multiple
print and digital sources; assess the credibility	print and digital sources, using search terms	print and digital sources, using search terms
of each source; and quote or paraphrase the	effectively; assess the credibility and accuracy	effectively; assess the credibility and accuracy
data and conclusions of others while avoiding	of each source; and quote or paraphrase the	of each source; and quote or paraphrase the

Grade 6 students:	Grade 7 students:	Grade 8 students:
plagiarism and providing basic bibliographic information for sources.  9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply grade 6 Reading standards to literature (e.g., "Compare and contrast texts in different forms or genres (e.g., stories and poems; historical novels and fantasy stories) in terms of their approaches to similar themes and topics."). b. Apply grade 6 Reading standards to literary nonfiction (e.g., "Trace and evaluate the argument and specific claims in a text, distinguishing claims that are supported by reasons and evidence from claims that are not").	data and conclusions of others while avoiding plagiarism and following a standard format for citation.  9. Draw evidence from literary or informational texts to support analysis, reflection, and research.  a. Apply grade 7 Reading standards to literature (e.g., "Compare and contrast a fictional portrayal of a time, place, or character and a historical account of the same period as a means of understanding how authors of fiction use or alter history.").  b. Apply grade 7 Reading standards to literary nonfiction (e.g. "Trace and evaluate the argument and specific claims in a text, assessing whether the reasoning is sound and the evidence is relevant and sufficient to support the claims.").	data and conclusions of others while avoiding plagiarism and following a standard format for citation.  9. Draw evidence from literary or informational texts to support analysis, reflection, and research.  a. Apply grade 8 Reading standards to literature (e.g., "Analyze how a modern work of fiction draws on themes, patterns of events, or character types from myths, traditional stories, or religious works such as the Bible, including describing how the material is rendered new.").  b. Apply grade 8 Reading standards to literary nonfiction (e.g., "Delineate and evaluate the argument and specific claims in a text [e.g., identifies bias and propaganda techniques, well-supported logical arguments], assessing whether the reasoning is sound and the evidence is relevant and sufficient; recognize
Range of Writing	Range of Writing	when irrelevant evidence is introduced.").  Range of Writing
10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.

### Writing Standards 6-12

The anchor standards and high school grade-specific standards work in tandem to define expectations—the former providing broad standards, the latter providing additional specificity.

Grades 9-10 students:	Grades 11-12 students:
Text Types and Purposes	Text Types and Purposes
1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.  a. Introduce precise claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that establishes clear relationships among claim(s), counterclaims, reasons, and evidence.  b. Develop claim(s) and counterclaims fairly, supplying evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level and concerns.  c. Use words, phrases, and clauses to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims.  d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing.  e. Provide a concluding statement or section that follows from and supports the argument presented.	1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.  a. Introduce precise, knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences claim(s), counterclaims, reasons, and evidence.  b. Develop claim(s) and counterclaims fairly and thoroughly, supplying the most relevant evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level, concerns, values, and possible biases.  c. Use words, phrases, and clauses as well as varied syntax to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims.  d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing.  e. Provide a concluding statement or section that follows from and
2. Write informative/explanatory texts to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.  a. Introduce a topic; organize complex ideas, concepts, and information to make important connections and distinctions; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension.  b. Develop the topic with well-chosen, relevant, and sufficient facts, extended definitions, concrete details, quotations, or other	supports the argument presented.  2. Write informative/explanatory texts to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.  a. Introduce a topic; organize complex ideas, concepts, and information so that each new element builds on that which precedes it to create a unified whole; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension.

Grades 9-10 students:	Grades 11-12 students:
information and examples appropriate to the audience's knowledge of the topic.	b. Develop the topic thoroughly by selecting the most significant and relevant facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic.
c. Use appropriate and varied transitions to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts. d. Use precise language and domain-specific vocabulary to manage the complexity of the topic. e. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. f. Provide a concluding statement or section that follows from and supports the information or explanation presented (e.g., articulating implications or the significance of the topic).	c. Use appropriate and varied transitions and syntax to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts.  d. Use precise language, domain-specific vocabulary, and techniques such as metaphor, simile, and analogy to manage the complexity of the topic.  e. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing.  f. Provide a concluding statement or section that follows from and supports the information or explanation presented (e.g., articulating implications or the significance of the topic).
3. Use narrative writing to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences.  a. Engage and orient the reader by setting out a problem, situation, or observation, establishing one or multiple point(s) of view, and	3. Use narrative writing to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences.  a. Engage and orient the reader by setting out a problem, situation, or observation and its significance, establishing one or multiple point(s) of
<ul> <li>introducing a narrator and/or characters; create a smooth progression of experiences or events.</li> <li>b. Use narrative techniques, such as dialogue, pacing, description, reflection, and multiple plot lines, to develop experiences, events, and/or characters.</li> </ul>	view, and introducing a narrator and/or characters; create a smooth progression of experiences or events.  b. Use narrative techniques, such as dialogue, pacing, description, reflection, and multiple plot lines, to develop experiences, events, and/or characters.
<ul><li>c. Use a variety of techniques to sequence events so that they build on one another to create a coherent whole.</li><li>d. Use precise words and phrases, telling details, and sensory language to convey a vivid picture of the experiences, events, setting, and/or characters.</li></ul>	<ul> <li>c. Use a variety of techniques to sequence events so that they build on one another to create a coherent whole and build toward a particular tone and outcome (e.g., a sense of mystery, suspense, growth, or resolution).</li> <li>d. Use precise words and phrases, telling details, and sensory language</li> </ul>
e. Provide a conclusion that follows from and reflects on what is experienced, observed, or resolved over the course of the narrative.	to convey a vivid picture of the experiences, events, setting, and/or characters.

Grades 9-10 students:	Grades 11-12 students:
	e. Provide a conclusion that follows from and reflects on what is
	experienced, observed, or resolved over the course of the narrative.
Production and Distribution of Writing	Production and Distribution of Writing
4. Produce clear and coherent writing in which the development,	4. Produce clear and coherent writing in which the development,
organization, style, and features are appropriate to task, genre,	organization, style, and features are appropriate to task, genre,
purpose, and audience. (Grade-specific expectations for writing types	purpose, and audience. (Grade-specific expectations for writing types
are defined in standards 1–3 above.)	are defined in standards 1–3 above.)
5. Develop and strengthen writing as needed by planning, revising,	5. Develop and strengthen writing as needed by planning, revising,
editing, rewriting, or trying a new approach, focusing on addressing	editing, rewriting, or trying a new approach, focusing on addressing
what is most significant for a specific purpose and audience. (Editing	what is most significant for a specific purpose and audience. (Editing
for conventions should demonstrate command of Language standards	for conventions should demonstrate command of Language standards
1–3 up to and including grades 9–10.)	1–3 up to and including grades 11–12.)
<b>6.</b> Use technology, including the Internet, to produce, publish, and	<b>6.</b> Use technology, including the Internet, to produce, publish, and
update individual or shared writing products, taking advantage of	update individual or shared writing products in response to ongoing
technology's capacity to link to other information and to display	feedback, including new arguments or information.
information flexibly and dynamically.	
Research to Build and Present Knowledge	Research to Build and Present Knowledge
<b>7.</b> Conduct short as well as more sustained research projects to answer	<b>7.</b> Conduct short as well as more sustained research projects to answer
a question (including a self-generated question) or solve a problem;	a question (including a self-generated question) or solve a problem;
narrow or broaden the inquiry when appropriate; synthesize multiple	narrow or broaden the inquiry when appropriate; synthesize multiple
sources on the subject, demonstrating understanding of the subject	sources on the subject, demonstrating understanding of the subject
under investigation.	under investigation.
8. Gather relevant information from multiple authoritative print and	8. Gather relevant information from multiple authoritative print and
digital sources, using advanced searches effectively; assess the	digital sources, using advanced searches effectively; assess the
usefulness of each source in answering the research question;	strengths and limitations of each source in terms of the task, purpose,
integrate information into the text selectively to maintain the flow of	and audience; integrate information into the text selectively to
ideas, avoiding plagiarism and following a standard format for citation.	maintain the flow of ideas, avoiding plagiarism and overreliance on any
	one source and following a standard format for citation.
<b>9.</b> Draw evidence from literary or informational texts to support	<b>9.</b> Draw evidence from literary or informational texts to support
analysis, reflection, and research.	analysis, reflection, and research.
a. Apply grades 9–10 Reading standards to literature (e.g., "Analyze	a. Apply grades 11–12 Reading standards to literature (e.g.,
how an author draws on and transforms source material in a specific	"Demonstrate knowledge of eighteenth-, nineteenth- and early
	twentieth-century foundational works of American literature, including

Grades 9-10 students:	Grades 11-12 students:
work [e.g., how Shakespeare treats a theme or topic from Ovid or how a later author draws on a play by Shakespeare].").  b. Apply grades 9–10 Reading standards to literary nonfiction (e.g., "Delineate and evaluate the argument and specific claims in a text [e.g., bias and propaganda techniques, emotional effect of specific word choices and sentence structures, well-supported logical arguments], assessing whether the reasoning is valid and the evidence is relevant and sufficient; identify false statements and fallacious reasoning.").	how two or more texts from the same period treat similar themes or topics.").  b. Apply grades 11–12 Reading standards to literary nonfiction (e.g., "Delineate and evaluate the reasoning in seminal U.S. texts, including the application of constitutional principles and use of legal reasoning [e.g., in U.S. Supreme Court Case majority opinions and dissents] and the premises, purposes, and arguments in works of public advocacy [e.g., <i>The Federalist</i> , presidential addresses].").
Range of Writing	Range of Writing
10. Write routinely over extended time frames (time for research,	10. Write routinely over extended time frames (time for research,
reflection, and revision) and shorter time frames (a single sitting or a	reflection, and revision) and shorter time frames (a single sitting or a
day or two) for a range of tasks, purposes, and audiences.	day or two) for a range of tasks, purposes, and audiences.

## **Speaking and Listening Standards 6-12**

The following standards for grades 6 - 12 offer a focus for instruction in each year to help ensure that students gain adequate mastery of a range of skills and applications. Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.

Grade 6 students:	Grade 7 students:	Grade 8 students:
Comprehension and Collaboration	Comprehension and Collaboration	Comprehension and Collaboration
1. Engage effectively in a range of	1. Engage effectively in a range of	1. Engage effectively in a range of
collaborative discussions (one-on-one, in	collaborative discussions (one-on-one, in	collaborative discussions (one-on-one, in
groups, and teacher-led) with diverse	groups, and teacher-led) with diverse	groups, and teacher-led) with diverse
partners on grade 6 topics, texts, and issues,	partners on grade 7 topics, texts, and issues,	partners on grade 8 topics, texts, and issues,
building on others' ideas and expressing their	building on others' ideas and expressing their	building on others' ideas and expressing their
own clearly.	own clearly.	own clearly.
a. Come to discussions prepared, having read	a. Come to discussions prepared, having read	a. Come to discussions prepared, having read
or studied required material; explicitly draw	or researched material under study; explicitly	or researched material under study; explicitly
on that preparation by referring to evidence	draw on that preparation by referring to	draw on that preparation by referring to
on the topic, text, or issue to probe and	evidence on the topic, text, or issue to probe	evidence on the topic, text, or issue to probe
reflect on ideas under discussion.	and reflect on ideas under discussion.	and reflect on ideas under discussion.
b. Follow rules for collegial discussions (e.g.,	b. Follow rules for collegial discussions (e.g.,	b. Follow rules for collegial discussions (e.g.,
establishing norms: taking turns,	establishing norms: taking turns,	establishing norms: taking turns,
paraphrasing, respecting diverse viewpoints),	paraphrasing, respecting diverse viewpoints),	paraphrasing, respecting diverse viewpoints),
set specific goals and deadlines, and define	track progress toward specific goals and	and decision-making (e.g., coming to
individual roles as needed.	deadlines, and define individual roles as	consensus), track progress toward specific
c. Pose and respond to specific questions	needed.	goals and deadlines, and define individual
with elaboration and detail by making	c. Pose questions that elicit elaboration and	roles as needed.
comments that contribute to the topic, text,	respond to others' questions and comments	c. Pose questions that connect the ideas of
or issue under discussion.	with relevant observations and ideas that	several speakers and respond to others'
d. Review the key ideas expressed and	bring the discussion back on topic as needed.	questions and comments with relevant
demonstrate understanding of multiple	d. Acknowledge new information expressed	evidence, observations, and ideas.
perspectives through reflection and	by others and, when warranted, modify their	d. Acknowledge new information expressed
paraphrasing.	own views.	by others, and, when warranted, qualify or
		justify their own views in light of the evidence
		presented.
2. Interpret information presented in diverse	2. Analyze the main ideas and supporting	<b>2.</b> Analyze the purpose of information
media (included but not limited to podcasts)	details presented in diverse media and	presented in diverse media and formats (e.g.,

Grade 6 students:	Grade 7 students:	Grade 8 students:
Comprehension and Collaboration	Comprehension and Collaboration	Comprehension and Collaboration
and formats (e.g., visually, quantitatively/	formats (e.g., visually, quantitatively/data-	visually, quantitatively/data-related, orally)
data-related, orally) and explain how it	related, orally) and explain how the ideas	and evaluate the motives (e.g., social,
contributes to a topic, text, or issue under	clarify a topic, text, or issue under study.	commercial, political) behind its presentation.
study.		
3. Delineate a speaker's argument and	3. Delineate a speaker's argument and	3. Delineate a speaker's argument and
specific claims, distinguishing claims that are	specific claims, evaluating the soundness of	specific claims, evaluating the soundness of
supported by reasons and evidence from	the reasoning and the relevance and	the reasoning and relevance and sufficiency
claims that are not.	sufficiency of the evidence.	of the evidence and identifying when
		irrelevant evidence is introduced.
Presentation of Knowledge and Ideas	Presentation of Knowledge and Ideas	Presentation of Knowledge and Ideas
4. Present claims and findings, sequencing	4. Present claims and findings, emphasizing	4. Present claims and findings, emphasizing
ideas logically and using pertinent	salient points in a focused, coherent manner	salient points in a focused, coherent manner
descriptions, facts, and details to accentuate	with pertinent descriptions, facts, details, and	with relevant evidence, sound valid
main ideas or themes; use appropriate eye	examples; use appropriate eye contact,	reasoning, and well-chosen details; use
contact, adequate volume, and clear	adequate volume, and clear pronunciation.	appropriate eye contact, adequate volume,
pronunciation.		and clear pronunciation.
5. Include multimedia components (e.g.,	5. Include multimedia components and visual	5. Integrate multimedia and visual displays
graphics, images, music, sound) and visual	displays in presentations to clarify claims and	into presentations to clarify information,
displays in presentations to clarify	findings and emphasize salient points.	strengthen claims and evidence, and add
information.		interest.
<b>6.</b> Adapt speech to a variety of contexts and	<b>6.</b> Adapt speech to a variety of contexts and	<b>6.</b> Adapt speech to a variety of contexts and
tasks, demonstrating command of formal	tasks, demonstrating command of formal	tasks, demonstrating command of formal
English when indicated or appropriate. (See	English when indicated or appropriate. (See	English when indicated or appropriate. (See
grade 6 Language standards 1 and 3 for	grade 7 Language standards 1 and 3 for	grade 8 Language standards 1 and 3 for
specific expectations.)	specific expectations.)	specific expectations.)

### **Speaking and Listening Standards 6-12**

The anchor standards and high school grade-specific standards work in tandem to define expectations—the former providing broad standards, the latter providing additional specificity.

Grades 9-10 students:	Grades 11-12 students:
Comprehension and Collaboration	Comprehension and Collaboration
1. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grades 9–10 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.  a. Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well-reasoned exchange of ideas.  b. Work with peers to set rules for collegial discussions and decision-making (e.g., informal consensus, taking votes on key issues, presentation of alternate views), clear goals and deadlines, and individual roles as needed.  c. Propel conversations by posing and responding to questions that relate the current discussion to broader themes or larger ideas; actively incorporate others into the discussion; and clarify, verify, or challenge ideas and conclusions.  d. Respond thoughtfully to diverse perspectives, summarize points of agreement and disagreement, and, when warranted, qualify or justify their own views and understanding and make new connections in light	1. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.  a. Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well-reasoned exchange of ideas.  b. Work with peers to promote civil, democratic discussions and decision-making, set clear goals and deadlines, and establish individual roles as needed.  c. Propel conversations by posing and responding to questions that probe reasoning and evidence; ensure a hearing for a full range of positions on a topic or issue; clarify, verify, or challenge ideas and conclusions; and promote divergent and creative perspectives.  d. Respond thoughtfully to diverse perspectives or arguments; synthesize comments, claims, and evidence made on all sides of an issue; resolve contradictions when possible; and determine what additional information or research is required to deepen the
of the evidence and reasoning presented.  2. Integrate multiple sources of information presented in diverse	investigation or complete the task.  2. Integrate multiple sources of information presented in diverse
media or formats (e.g., visually, quantitatively, orally) evaluating the credibility and accuracy of each source and noting any discrepancies	formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility
among data or information.	and accuracy of each source and noting any discrepancies among the data or information.
3. Identify and evaluate a speaker's point of view, reasoning, and use	3. Identify and evaluate a speaker's point of view, reasoning, and use
of evidence and rhetoric, identifying any fallacious reasoning or exaggerated or distorted evidence.	of evidence and rhetoric, assessing the stance, premises, links among ideas, word choice, points of emphasis, and tone used.
chaggerated of distorted evidence.	ideas, word choice, points of emphasis, and tone used.

### **Speaking and Listening Standards 6-12**

Grades 9-10 students:	Grades 11-12 students:
Presentation of Knowledge and Ideas	Presentation of Knowledge and Ideas
4. Present information, findings, and supporting evidence clearly,	4. Present information, findings, and supporting evidence, conveying a
concisely, and logically such that listeners can follow the line of	clear and distinct perspective, such that listeners can follow the line of
reasoning and the organization, development, substance, and style are	reasoning, alternative or opposing perspectives are addressed, and the
appropriate to purpose, audience, and task.	organization, development, substance, and style are appropriate to
	purpose, audience, and a range or formal and informal tasks.
5. Make strategic use of digital media (e.g., textual, graphical, audio,	5. Make strategic use of digital media (e.g., textual, graphical, audio,
visual, and interactive elements) in presentations to enhance	visual, and interactive elements) in presentations to enhance
understanding of findings, reasoning, and evidence and to add	understanding of findings, reasoning, and evidence and to add
interest.	interest.
6. Adapt speech to a variety of contexts and tasks, demonstrating	<b>6.</b> Adapt speech to a variety of contexts and tasks, demonstrating a
command of formal English when indicated or appropriate and	command of formal English when indicated or appropriate and
addressing intended audience needs and knowledge level. (See grades	addressing intended audience needs and knowledge level. (See grades
9–10 Language standards 1 and 3 for specific expectations.)	11–12 Language standards 1 and 3 for specific expectations.)

The following standards for grades 6-12 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades. Beginning in grade 3, skills and understandings that are particularly likely to require continued attention in higher grades as they are applied to increasingly sophisticated writing and speaking are marked with an asterisk (\*). For a complete list see the table in the Introduction.

Grade 6 students:	Grade 7 students:	Grade 8 students:
Conventions of Standard English	Conventions of Standard English	Conventions of Standard English
1. Demonstrate command of the conventions	1. Demonstrate command of the conventions	1. Demonstrate command of the conventions
of standard English grammar and usage when	of standard English grammar and usage when	of standard English grammar and usage when
writing or speaking.	writing or speaking.	writing or speaking.
a. Ensure that pronouns are in the proper	a. Explain the function of phrases and clauses	a. Explain the function of verbals (gerunds,
case (subjective, objective, possessive).	in general and their function in specific	participles, infinitives) in general and their
b. Use intensive pronouns (e.g., myself,	sentences in order to apply the conventions	function in particular sentences in order to
ourselves).	of English.	apply the conventions of English.
c. Recognize and correct inappropriate shifts	b. Choose among simple, compound,	b. Form and use verbs in the active and
in pronoun number and person.*	complex, and compound-complex sentences	passive voice.
d. Recognize and correct vague pronouns (i.e.,	to signal differing relationships among ideas.	c. Form and use verbs in the indicative,
ones with unclear or ambiguous	c. Place phrases and clauses within a	imperative, interrogative, conditional, and
antecedents).*	sentence, recognizing and correcting	subjunctive mood.
e. Recognize variations from standard English	misplaced and dangling modifiers.*	d. Recognize and correct inappropriate shifts
in their own and others' writing and speaking,		in verb voice and mood.*
and identify and use strategies to improve		
expression in conventional language.*		
2. Demonstrate command of the conventions	2. Demonstrate command of the conventions	2. Demonstrate command of the conventions
of standard English capitalization,	of standard English capitalization,	of standard English capitalization,
punctuation, and spelling when writing.	punctuation, and spelling when writing.	punctuation, and spelling when writing.
a. Use punctuation (commas, parentheses,	a. Use a comma to separate coordinate	a. Use punctuation (comma, ellipsis, dash) to
dashes) to set off nonrestrictive/parenthetical	adjectives (e.g., It was a fascinating,	indicate a pause or break.
elements.*	enjoyable movie.).	
b. Spell correctly.	b. Spell correctly.	b. Use an ellipsis to indicate an omission.
		c. Spell correctly.

### Language Standards 6-12

:	Grade 8 students:
iguage	Knowledge of Language
of language and its name of language and its name of writing, speaking, reading, ge that expresses ideas cisely, recognizing and ness and redundancy.*	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening.  a. Use verbs in the active and passive voice and in the conditional and subjunctive mood to achieve particular effects (e.g., emphasizing the actor or the action; expressing uncertainty or describing a state contrary to fact).
sition and Use	Vocabulary Acquisition and Use
larify the meaning of ltiple-meaning words and grade 7 reading and flexibly from a range of ming of unfamiliar words by of word structure, base words, common gins), context (e.g., the f a sentence or paragraph; a function in a sentence) as ning of a word or phrase. Grade-appropriate Greek or oots as clues to the d (e.g., belligerent, bellicose, and specialized reference ctionaries, glossaries, and a print and digital, to find	4. Determine or clarify the meaning of unknown and multiple-meaning words or phrases based on grade 8 reading and content, choosing flexibly from a range of strategies.  a. Determine meanings of unfamiliar words by using knowledge of word structure, (prefixes/suffixes, base words, common roots, or word origins), context (e.g., the overall meaning of a sentence or paragraph; a word's position or function in a sentence), knowledge of language structure including using context clues and prior knowledge.  b. Use common, grade-appropriate Greek or Latin affixes and roots as clues to the meaning of a word (e.g., precede, recede, secede).  c. Consult general and specialized reference materials (e.g., dictionaries, glossaries, and
ctionar h print	

Grade 6 students:	Grade 7 students:	Grade 8 students:
and digital, to find the pronunciation of a word or determine or clarify its precise meaning or its part of speech. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).	clarify its precise meaning or its part of speech. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).	clarify its precise meaning or its part of speech. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).
5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret the intent or meaning of figures of speech (e.g., personification, metaphors, alliteration) as used in context. b. Use the relationship between particular words (e.g., cause/effect, part/whole, item/category) to better understand each of the words. c. Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., stingy, scrimping, economical, unwasteful, thrifty).	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings.  a. Interpret the intent or meaning of figures of speech (e.g., literary, religious, and mythological allusions; euphemisms) as used in context.  b. Use the relationship between particular words (e.g., synonym/antonym, analogy) to better understand each of the words.  c. Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., refined, respectful, polite, diplomatic, condescending).	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret the intent or meaning of figures of speech (e.g., verbal irony, puns, mixed metaphor) as used in context. b. Use the relationship between particular words (e.g., synonyms/antonyms, analogies) to better understand each of the words. c. Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., bullheaded, willful, firm, persistent, resolute).
<b>6.</b> Acquire and accurately use gradeappropriate general academic and domainspecific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression.	6. Acquire and accurately use grade- appropriate general academic and domain- specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression.	<b>6.</b> Acquire and accurately use gradeappropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression.

### Language Standards 6-12

The anchor standards and high school grade-specific standards work in tandem to define readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 9-10 students:	Grades 11-12 students:
Conventions of Standard English	Conventions of Standard English
1. Demonstrate command of the conventions of standard English	1. Demonstrate command of the conventions of standard English
grammar and usage when writing or speaking.	grammar and usage when writing or speaking.
a. Use parallel structure.*	a. Apply the understanding that usage is a matter of convention, can
b. Use various types of phrases (noun, verb, adjectival, adverbial,	change over time, and is sometimes contested.
participial, prepositional, absolute) and clauses (independent,	b. Resolve issues of complex or contested usage, consulting
dependent; noun, relative, adverbial) to convey specific meanings and	references (e.g., Merriam-Webster's Dictionary of English Usage,
add variety and interest to writing or presentations.	Garner's Modern American Usage) as needed.
2. Demonstrate command of the conventions of standard English	2. Demonstrate command of the conventions of standard English
capitalization, punctuation, and spelling when writing.	capitalization, punctuation, and spelling when writing.
a. Use a semicolon (and perhaps a conjunctive adverb) to link two or	a. Observe hyphenation conventions.
more closely related independent clauses.	b. Spell correctly.
b. Use a colon to introduce a list or quotation.	
c. Spell correctly.	
Knowledge of Language	Knowledge of Language
3. Apply knowledge of language to understand how language	3. Apply knowledge of language to understand how language
functions in different contexts, to make effective choices for meaning	functions in different contexts, to make effective choices for meaning
or style, and to comprehend more fully when reading or listening.	or style, and to comprehend more fully when reading or listening.
a. Write and edit work so that it conforms to the guidelines in a style	a. Vary syntax for effect, consulting references (e.g., Tufte's Artful
manual (e.g., MLA Handbook, Turabian's Manual for Writers)	Sentences) for guidance as needed; apply an understanding of syntax
appropriate for the discipline and writing type.	to the study of complex texts when reading.

Grades 9-10 students:	Grades 11-12 students:
Vocabulary Acquisition and Use	Vocabulary Acquisition and Use
4. Determine or clarify the meaning of unknown and multiple-	4. Determine or clarify the meaning of unknown and multiple-
meaning words and phrases based on grades 9-10 reading and	meaning words and phrases based on grades 11-12 reading and
content, choosing flexibly from a range of strategies.	content, choosing flexibly from a range of strategies.
a. Determine meanings of unfamiliar words by using knowledge of	a. Determine meanings of unfamiliar words by using knowledge of
derivational roots and affixes, including cultural derivations (e.g., the	derivational roots and affixes, including cultural derivations (e.g., the
root of photography and photosynthesis; kayak), context (e.g., the	root of photography and photosynthesis; kayak), context (e.g., the
overall meaning of a sentence, paragraph, or text; a word's position	overall meaning of a sentence, paragraph, or text; a word's position
or function in a sentence), dialectical English (e.g., Huck Finn),	or function in a sentence), dialectical English (e.g., Huck Finn),
idiomatic expressions (e.g., "it drives me up a wall") as clues to the	idiomatic expressions (e.g., "it drives me up a wall") as clues to the
meaning of a word or phrase.	meaning of a word or phrase.
b. Identify and correctly use patterns of word changes that indicate	b. Identify and correctly use patterns of word changes that indicate
different meanings or parts of speech (e.g., analyze, analysis,	different meanings or parts of speech (e.g., conceive, conception,
analytical; advocate, advocacy).	conceivable).
c. Consult general and specialized reference materials (e.g.,	c. Consult general and specialized reference materials (e.g.,
dictionaries, glossaries, thesauruses), both print and digital, to find	dictionaries, glossaries, thesauruses), both print and digital, to find
the pronunciation of a word or determine or clarify its precise	the pronunciation of a word or determine or clarify its precise
meaning, its part of speech, or its etymology.	meaning, its part of speech, its etymology, or its standard usage.
d. Verify the preliminary determination of the meaning of a word or	d. Verify the preliminary determination of the meaning of a word or
phrase (e.g., by checking the inferred meaning in context or in a dictionary).	phrase (e.g., by checking the inferred meaning in context or in a dictionary).
<b>5.</b> Demonstrate understanding of figurative language, word	5. Demonstrate understanding of figurative language, word
relationships, and nuances in word meanings.	relationships, and nuances in word meanings.
a. Interpret figures of speech (e.g., euphemism, oxymoron) in	a. Interpret figures of speech (e.g., hyperbole, paradox) in context
context and analyze their role in the text.	and analyze their role in the text.
b. Analyze nuances in the meaning of words with similar denotation	b. Analyze nuances in the meaning of words with similar denotations
(definition) or determine the meaning of analogies.	(definition) or determine the meaning of analogies.
<b>6.</b> Acquire and use accurately general academic and domain-specific	<b>6.</b> Acquire and use accurately general academic and domain-specific
words and phrases, sufficient for reading, writing, speaking, and	words and phrases, sufficient for reading, writing, speaking, and
listening at the college and career readiness level; demonstrate	listening at the college and career readiness level; demonstrate
independence in gathering vocabulary knowledge when considering	independence in gathering vocabulary knowledge when considering

a word or phrase important to comprehension or expression.

a word or phrase important to comprehension or expression.

Standards for Literacy in History/Social Studies, Science, and Technical Subjects 6-12

### **Reading Standards for Literacy in History/Social Studies 6-12**

The standards below begin in grade 6; standards for K-5 reading in history/social studies, science and technical subjects are integrated in the K-5 Reading standards. The Alaska Reading Anchor Standards and high school standards in literacy work in tandem to define college and career readiness expectations – the former providing broad standards, the latter providing additional specificity.

Grades 6-8 students:	Grades 9-10 students:	Grades 11-12 students:
Key Ideas and Details	Key Ideas and Details	Key Ideas and Details
<b>1.</b> Cite specific textual evidence to support analysis of primary and secondary sources.	<b>1.</b> Cite specific textual evidence to support analysis of primary and secondary sources, attending to such features as the date and origin of the information.	1. Cite specific textual evidence to support analysis of primary and secondary sources, connecting insights gained from specific details to an understanding of the text as a whole.
<b>2.</b> Determine the central ideas or information of a primary or secondary source; provide an accurate summary of the source distinct from prior knowledge or opinions.	<b>2.</b> Determine the central ideas or information of a primary or secondary source; provide accurate summary of how key events or ideas develop over the course of the text.	2. Determine the central ideas or information of a primary or secondary source; provide accurate summary that makes clear the relationships among the key details and ideas.
<b>3.</b> Identify key steps in a text's description of a process related to history/social studies (e.g., how a bill becomes law, how interest rates are raised or lowered).	<b>3.</b> Analyze in detail a series of events described in a text; determine whether earlier events caused later ones or simply preceded them.	<b>3.</b> Evaluate various explanations for actions or events and determine which explanation best accords with textual evidence, acknowledging where the text leaves matters uncertain.
Craft and Structure	Craft and Structure	Craft and Structure
<b>4.</b> Determine the meaning of words and phrases as they are used in a text, including vocabulary specific to domains related to history/social studies.	<b>4.</b> Determine the meaning of words and phrases as they are used in a text, including vocabulary describing political, social, or economic aspects of history/social studies.	<b>4.</b> Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of specific word choices on meaning and tone, including analogies or allusions to other texts.
<b>5.</b> Describe how a text presents information (e.g., sequentially, comparatively, causally)	<b>5.</b> Analyze how a text uses structure to emphasize key points or advance an explanation or analysis.	<b>5.</b> Analyze in detail how a complex primary source is structured, including how key sentences, paragraphs, and larger portions of the text contribute to the whole.

Grades 6-8 students:	Grades 9-10 students:	Grades 11-12 students:
<b>6.</b> Identify aspects of a text that reveal an author's point of view or purpose (e.g., loaded language, inclusion or avoidance of particular facts).	<b>6.</b> Compare the point of view of two or more authors for how they treat the same or similar topics, including which details they include and emphasize in their respective accounts.	<b>6.</b> Evaluate authors' differing points of view on the same historical event or issue by assessing the authors' claims, reasoning, and evidence.
Integration of Knowledge and Ideas	Integration of Knowledge and Ideas	Integration of Knowledge and Ideas
<b>7.</b> Integrate visual information (e.g., in charts, graphs, photographs, videos, or maps) with other information in print and digital texts.	<b>7.</b> Integrate quantitative or technical analysis (e.g., charts, research data) with qualitative analysis in print or digital texts.	7. Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, as well as in words) in order to address a question or solve a problem.
8. Distinguish among fact, opinion, and	<b>8.</b> Assess the extent to which the reasoning	8. Evaluate an author's premises, claims, and
reasoned judgment in a text.	and evidence in a text support the author's claim.	evidence by corroborating or challenging them with other information.
9. Analyze the relationship between a	9. Compare and contrast treatments of the	9. Integrate information from diverse sources,
primary and secondary source on the same	same topic in several primary and secondary	both primary and secondary, into a coherent
topic.	sources.	understanding of an idea or event, noting
		discrepancies among sources.
Range of Reading and Level of Text	Range of Reading and Level of Text	Range of Reading and Level of Text
Complexity	Complexity	Complexity
<b>10.</b> By the end of grade 8, read and	<b>10.</b> By the end of grade 10, read and	<b>10.</b> By the end of grade 12, read and
comprehend history/social studies texts in	comprehend history/social studies texts in	comprehend history/social studies texts in
the grades 6-8 text complexity band	the grades 9-10 text complexity band	the grades 11-12 text complexity band
independently and proficiently.	independently and proficiently.	independently and proficiently.

# **Reading Standards for Literacy in Science and Technical Subjects 6-12**

Grades 6-8 students:	Grades 9-10 students:	Grades 11-12 students:
Key Ideas and Details	Key Ideas and Details	Key Ideas and Details
1. Cite specific textual evidence to support	1. Cite specific textual evidence to support	1. Cite specific textual evidence to support
analysis of science and technical texts.	analysis of science and technical texts,	analysis of science and technical texts,
	attending to the precise details of	attending to important distinctions the
	explanations or descriptions.	author makes and to any gaps or
		inconsistencies in the account.
<b>2.</b> Determine the central ideas or conclusions	2. Determine the central ideas or conclusions	2. Determine the central ideas or conclusions
of a text; provide an accurate summary of the	of a text; trace the text's explanation or	of a text; summarize complex concepts,
text distinct from prior knowledge or	depiction of a complex process,	processes, or information presented in a text
opinions.	phenomenon, or concept; provide an	by paraphrasing them in simpler but still
	accurate summary of the text.	accurate terms.
3. Follow precisely a multistep procedure	3. Follow precisely a complex multistep	3. Follow precisely a complex multistep
when carrying out experiments, taking	procedure when carrying out experiments,	procedure when carrying out experiments,
measurements, or performing technical tasks.	taking measurements, or performing technical	taking measurements, or performing
	tasks, attending to special cases or exceptions	technical tasks; analyze the specific results
	defined in the text.	based on explanations in the text.
Craft and Structure	Craft and Structure	Craft and Structure
<b>4.</b> Determine the meaning of symbols, key	<b>4.</b> Determine the meaning of symbols, key	<b>4.</b> Determine the meaning of symbols, key
terms, and other domain-specific words and	terms, and other domain-specific words and	terms, and other domain-specific words and
phrases as they are used in a specific scientific	phrases as they are used in a specific scientific	phrases as they are used in a specific scientific
or technical context relevant to grades 6-8	or technical context relevant to grades 9-10	or technical context relevant to grades 11-12
texts and topics.	texts and topics.	texts and topics.
<b>5.</b> Analyze the structure an author uses to	<b>5.</b> Analyze the structure of the relationships	5. Analyze how the text structures
organize a text, including how the major	among concepts in a text, including	information or ideas into categories or
sections contribute to the whole and to an	relationships among key terms (e.g., force,	hierarchies, demonstrating understanding of
understanding of the topic.	friction, reaction force, energy).	the information or ideas.
<b>6.</b> Analysis the author's purpose in providing	<b>6.</b> Analyze the author's purpose in providing	<b>6.</b> Analyze the author's purpose in providing
an explanation, describing a procedure, or	an explanation, describing a p0rocedure, or	an explanation, describing a procedure, or
discussing an experiment in a text.	discussing an experiment in a text, defining	discussing an experiment in a text, identifying
	the question the author seeks to address.	important issues that remain unresolved.

### Reading Standards for Literacy in Science and Technical Subjects 6-12

Grades 6-8 students:	Grades 9-10 students:	Grades 11-12 students:
Integration of Knowledge and Ideas	Integration of Knowledge and Ideas	Integration of Knowledge and Ideas
7. Integrate quantitative or technical	7. Translate quantitative or technical	7. Integrate and evaluate multiple sources of
information expressed in words in a text with	information expressed in words in a text into	information presented in diverse formats and
a version of that information expressed	visual from (e.g., a table or chart) and	media (e.g., quantitative data, video,
visually (e.g., in a flowchart, diagram, model,	translate information expressed visually or	multimedia) in order to address a question or
graph, or table).	mathematically (e.g., in an equation) into	solve a problem.
	words.	
8. Distinguish among facts, reasoned	8. Assess the extent to which the reasoning	8. Evaluate the hypotheses, data, analysis,
judgment based on research findings, and	and evidence in a text support the author's	and conclusions in a science or technical text,
speculation in a text.	claim or a recommendation for solving a	verifying the data when possible and
	scientific or technical problem.	corroborating or challenging conclusions with
		other sources of information.
9. Compare and contrast the information	9. Compare and contrast findings presented	<b>9.</b> Synthesize information from a range of
gained from experiments, simulations, video,	in a text to those from other sources	sources (e.g., texts, experiments, simulations)
or multimedia sources with that gained from	(including their own experiments), noting	into a coherent understanding of a process,
reading a text on the same topic.	when the findings support or contradict	phenomenon or concept, resolving conflicting
	previous explanations or accounts.	information when possible.
Range of Reading and Level of Text	Range of Reading and Level of Text	Range of Reading and Level of Text
Complexity	Complexity	Complexity
<b>10.</b> By the end of grade 8, read and	<b>10.</b> By the end of grade 10, read and	<b>10.</b> By the end of grade 12, read and
comprehend science/technical texts in the	comprehend science/technical texts in the	comprehend science/technical texts in the
grades 6-8 text complexity band	grades 9-10 text complexity band	grades 11-12 text complexity band
independently and proficiently.	independently and proficiently.	independently and proficiently.

## Writing Standards for Literacy in History/Social Studies, Science and Technical Subjects 6-12

The standards below begin in grade 6; standards for K-5 writing in history/social studies, science, and technical subjects are integrated into the K-5 writing standards. The Alaska Writing Anchor Standards and high school standards in literacy work in tandem to define college and career readiness expectations – the former providing broad standards, the latter providing additional specificity.

discipline- discipline- discipline- specific content. a. Introduce precise claim(s), knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences the claim(s) counterclaims reasons and
specific content. a. Introduce precise claim(s), knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences the
claims fairly, reach while evidence. b. Develop claim(s) and counterclaims fairly and thoroughly, supplying the most relevant data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form that anticipates the audience's knowledge level, concerns, values, and possible biases. c. Use words, phrases, and clauses as well as varied syntax to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons between reasons and evidence, and between claim(s) and counterclaims. d. Establish and maintain a formal style and objective tone while attending to the norms
and conventions of the discipline in which
t

Grades 6-8 students:	Grades 9-10 students:	Grades 11-12 students:
		e. Provide a concluding statement or section that follows from and supports the argument presented.
<ul> <li>2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/experiments, or technical processes.</li> <li>a. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories as appropriate to achieving purpose; include formatting (e.g., headings) graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension.</li> </ul>	2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/experiments, or technical processes.  a. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information, using strategies such as definition, classification, comparison/contrast, and cause/effect; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension.	2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/experiments, or technical processes.  a. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension.
b. Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples. c. Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Establish and maintain a formal style and objective tone. f. Provide a concluding statement or section that follows from and supports the information or explanation presented.	b. Develop the topic with well-chosen, relevant, and sufficient facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic. c. Use varied transitions and sentence structures to link the major sections of the text, create cohesion, and clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to manage the complexity of the topic and convey a style appropriate to the discipline and context as well as to the expertise of likely readers. e. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing.	b. Develop the topic thoroughly by selecting the most significant and relevant facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic. c. Use varied transitions and sentence structures to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts. d. Use precise language and domain-specific vocabulary and techniques such as metaphor, simile, and analogy to manage the complexity of the topic; convey a knowledgeable stance in a style that responds to the discipline and context as well as to the expertise of likely readers.

Grades 6-8 students:	Grades 9-10 students:	Grades 11-12 students:
	f. Provide a concluding statement or section that follows from and supports the information or explanation presented (e.g., articulating implications or the significance of the topic).	e. Provide a concluding statement or section that follows from and supports the information or explanation presented (e.g., articulating implications or the significance of the topic).
3. Not applicable as a separate requirement.	3. Not applicable as a separate requirement.	3. Not applicable as a separate requirement.

**Note:** Students' narrative skills continue to grow in these grades. The Standards require that students be able to incorporate narrative elements effectively into arguments and information/explanatory texts. In history/social studies, students must be able to incorporate narrative accounts into their analyses of individuals or events of historical import. In science and technical subjects, students must be able to write precise enough descriptions of the step-by-step procedures they use in their investigations or technical work that others can replicate them and (possibly) reach the same results.

Production and Distribution of Writing	Production and Distribution of Writing	Production and Distribution of Writing
4. Produce clear and coherent writing in	<b>4.</b> Produce clear and coherent writing in	4. Produce clear and coherent writing in
which the development, organization, and	which the development, organization, and	which the development, organization, and
style are appropriate to task, purpose, and	style are appropriate to task, purpose, and	style are appropriate to task, purpose, and
audience.	audience.	audience.
<b>5.</b> With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed.	<b>5.</b> Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.	<b>5.</b> Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on what is most significant for a specific purpose and audience.
<b>6.</b> Use technology, including the Internet, to produce and publish writing and present the relationships between information and ideas clearly and efficiently.	<b>6.</b> Use technology, including the Internet, to produce, publish, and update individual or shared writing products, taking advantage of technology's capacity to link to other information and to display information flexibly and dynamically.	<b>6.</b> Use technology, including the Internet, to produce, publish, and update individual or shared writing products in response to ongoing feedback, including new arguments or information.
Research to Build and Present Knowledge	Research to Build and Present Knowledge	Research to Build and Present Knowledge

Production and Distribution of Writing	Production and Distribution of Writing	Production and Distribution of Writing
7. Conduct short research projects to answer	7. Conduct short as well as more sustained	7. Conduct short as well as more sustained
a question (including a self-generated	research projects to answer a question	research projects to answer a question
question), drawing on several sources and	(including a self-generated question) or solve	(including a self-generated question) or solve
generating additional related, focused	a problem; narrow or broaden the inquiry	a problem; narrow or broaden the inquiry
questions that allow for multiple avenues of	when appropriate; synthesize multiple	when appropriate; synthesize multiple
exploration.	sources on the subject, demonstrating	sources on the subject, demonstrating
	understanding of the subject under	understanding of the subject under
	investigation.	investigation.
8. Gather relevant information from multiple	8. Gather relevant information from multiple	8. Gather relevant information from multiple
print and digital sources, using search terms	authoritative print and digital sources, using	authoritative print and digital sources, using
effectively; assess the credibility and accuracy	advanced searches effectively; assess the	advanced searches effectively; assess the
of each source; and quote or paraphrase the	usefulness of each source in answering the	strengths and limitations of each source in
data and conclusions of others while avoiding	research question; integrate information into	terms of the specific task, purpose, and
plagiarism and following a standard format	the text selectively to maintain the flow of	audience; integrate information into the text
for citation.	ideas, avoiding plagiarism and following a	selectively to maintain the flow of ideas,
	standard format for citation.	avoiding plagiarism and overreliance on any
		one source and following a standard format
		for citation.
<b>9.</b> Draw evidence from informational texts to	<b>9.</b> Draw evidence from informational texts to	<b>9.</b> Draw evidence from informational texts to
support analysis, reflection, and research.	support analysis, reflection, and research.	support analysis, reflection, and research.
Range of Writing	Range of Writing	Range of Writing
10. Write routinely over extended time	10. Write routinely over extended time	10. Write routinely over extended time
frames (time for reflection and revision) and	frames (time for reflection and revision) and	frames (time for reflection and revision) and
shorter time frames (a single sitting or a day	shorter time frames (a single sitting or a day	shorter time frames (a single sitting or a day
or two) for a range of discipline-specific tasks,	or two) for a range of discipline-specific tasks,	or two) for a range of discipline-specific tasks,
purposes, and audiences.	purposes, and audiences.	purposes, and audiences.

# **Glossary for English/Language Arts Standards**

#### author's purpose

Purpose is the goal or objective the author is trying to accomplish; the intention or reason for writing a text; for example: to persuade, to entertain, to describe, to explain.

#### collegial

Marked by camaraderie among colleagues.

#### domain-specific words and phrases

Vocabulary specific to a particular field of study: analogous to tier-three words.

#### expository text

Nonfiction, factual prose written to explain or convey information.

#### fallacious

Embodying a fallacy.

#### figurative language

Language that moves beyond the literal in which a figure of speech is used to heighten the meaning; for example: hyperbole, metaphor, personification, or simile.

#### general academic words and phrases

Vocabulary common to written texts but not commonly a part of speech.

#### informational text

Explanatory writing that serves one or more closely related purposes: to increase readers' knowledge of a subject, to help readers better understand a procedure or process, or to provide readers with an enhanced comprehension of a concept. Informational/explanatory writing conveys information accurately.

#### literary text

This category is used to classify literary works, usually by form, technique, or content: novel, essay, short story, comedy, epic.

#### narrative writing

Writing that conveys experience, either real or imaginary, and uses time as its deep structure. It can be used for many purposes: inform, instruct, persuade, or entertain. Narratives take many forms: creative fictional stories, memoirs, anecdotes, and autobiographies.

#### persuasive writing

The purpose is to convince the reader of an idea, to change the reader's mind, or to move the reader to action: letters to the editor, political pamphlets.

#### register

A variety of language used for a specific purpose or in a particular social setting; appropriate register depends upon the audience, purpose, topic, and location; for example: a different type of language may be used for speaking to friends than to teachers.

#### scaffolding

Temporary guidance or assistance provided to a student by a teacher, another adult, or a more capable peer, enabling the student to perform a task he or she otherwise would not be able to do alone, with the goal of fostering the student's capacity to perform the task independently.

#### technical text

Practical, accurate, and precise communication of specialized information that imparts facts, persuades with facts, or analyzes data or problems. The format is clear and efficient, often including headings, itemized lists with bullets, diagrams, or tables: instruction manual, scientific article, sales letter, letter of recommendation, proposal, or memo.

# **Alaska Mathematics Standards**

#### **Introduction to Mathematics Standards**

The mathematics standards prepare Alaska students to be competitive on the national and world stage. These standards are a set of specific, rigorous expectations that build students' conceptual understanding, mathematical language, and application of processes and procedures coherently from one grade to the next so all students will be prepared for post-secondary experiences. The focus areas for each grade level and each conceptual category narrative establish a depth of knowledge as opposed to a breadth of knowledge across multiple standards in each grade level or content area.

The standards for mathematics stress both conceptual understanding and procedural skills to ensure students learn and can apply the critical information needed to succeed at each level.

- In kindergarten, the standards follow successful international models and recommendations by focusing kindergarten work on the number core: learning how numbers correspond to quantities, and learning how to put numbers together and take them apart (the beginnings of addition and subtraction).
- The K-5 standards provide students with a solid foundation in whole numbers, addition, subtraction, multiplication, division, fractions and decimals--which help young students build the foundation to successfully apply more demanding math concepts and procedures and move into applications.
- Having built a strong foundation in K-5, students can do hands-on learning in geometry, algebra and probability and statistics. Students who have completed 7th grade and mastered the content and skills through the 7th grade will be well-prepared for algebra in grade 8. The middle school standards are robust and provide a coherent and rich preparation for high school mathematics.
- The high school standards set a rigorous definition of readiness by helping students develop a depth of understanding and ability to apply mathematics to novel situations, as college students and employees regularly do.

## **Organization of Mathematics Standards**

The Alaska Mathematics Standards define what students should understand and be able to do in their study of mathematics. Teachers ensure students achieve standards by using a variety of instructional strategies based on their students' needs.

The standards are divided into two areas of equal importance:

- 1. **The Standards for Mathematical Practice** are embedded at every grade level to establish habits of mind that will empower students to become mathematically literate. Instructional approaches that promote students' development of the Practices are critical to procedural fluency in mathematics.
- 2. **The Standards for Mathematical Content** are grade-level specific in kindergarten through grade 8. The high school content is organized by conceptual category. Taken together, the K-12 standards provide a scaffold that allows students to become increasingly more proficient in understanding and using mathematics. There is a gradual, steady progression leading to college and career readiness by the time students graduate from high school.

Each grade-level is supported with the inclusion of an Instructional Focus section. The Instructional Focus guides teachers toward the critical areas of emphasis. Each high school Conceptual Category includes a narrative that also guides teachers' instruction.

#### The Standards for Mathematical Practice

These eight standards bring the complexities of the world into focus and give schema for grappling with authentic and meaningful problems. The practice standards define experiences that build understanding of mathematics and ways of thinking through which students develop, apply, and assess their knowledge.

Algorithmic knowledge is no longer sufficient when preparing our students to become globally competitive. The knowledge of good practitioners goes beyond algorithmic learning and allows them to picture the problem and the many roads that may lead to a solution. They realize that mathematics is applicable outside of the classroom and are confident in their ability to apply mathematical concepts to all aspects of life. The Standards of Mathematical Practice allow students to deepen their understandings of mathematical concepts and cultivates their autonomy as mathematically literate and informed citizens. Employing mathematics as a means of synthesizing complex concepts and making informed decisions is paramount to success in all post-secondary endeavors.

	Standards for Mathematical Practice								
Make sense of problems and persevere in solving them     5. Use appropriate tools strategically									
Reason abstractly and quantitatively     6. Attend to						to precision			
3. Construc	3. Construct viable arguments and critique the reasoning of others 7. Look for and make use of structure					:			
4. Model w	ith mather	natics				8. Look fo	r and express	regularity in r	epeated reasoning
Kindergarten	1	2	3	4	5	6	7	8	High School

Instruction around the Standards for Mathematical Practices is delivered across all grades K-12. For each Standard for Mathematical Practice, there are grade-span descriptors that are meant to help students, parents and educators determine how these might be demonstrated by students. Implementing the practices to meet the descriptors will involve strengthening current teaching practices.

#### The Standards for Mathematical Content

Each grade level in the K-8 standards is prefaced with an explanation of instructional focus areas for that grade level. Each conceptual category in the high school standards is prefaced with an explanation of the implication of that category to a student's mastery of mathematics. Specific modeling standards appear throughout the high school standards as indicated by an asterisk (\*).

Additional mathematic standards that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics are indicated by a plus symbol (+). The plus symbol indicates that the standard is not required for all students.

#### K-8 Mathematical Domains:

- 1. Counting and Cardinality CC
- 2. Operations and Algebraic Thinking OA
- 3. Number and Operations in Base Ten NBT
- 4. Measurement and Data MD
- 5. Number and Operations—Fractions NF
- 6. Geometry G
- 7. Ratios and Proportional Relationships RP
- 8. The Number System NS
- 9. Expressions and Equations EE
- 10. Functions F

### **High School Conceptual Categories:**

- 1. Number and Quantity N
- 2. Algebra A
- 3. Functions F
- 4. Modeling M
- 5. Geometry G
- 6. Statistics and Probability P

The standards for mathematics stress both conceptual understanding and procedural skills to ensure students learn and can apply the critical information needed to succeed at each level. This creates a learning progression where the mathematics learned in elementary school provides the foundation for the study of statistics, probability, ratio and proportion, geometry, and algebra in middle school. This is, in turn, the base upon which the knowledge needed for success in colleges and careers can be developed in high school.

The standards organization is not intended to convey the order of instruction nor the length of time to devote to the topics. In the standards, the clusters have been arranged in the grade span to show the continuum between grades. The following table outlines the progression of the content from kindergarten through high school.

				s	tandards fo	r Mathemat	ical Conten	t			
	Kindergarten	1	2	3	4	5	6	7	8	High Schoo	ol
1	Counting and Ca	ardinality				l	l	1		Number & Quantity	t
	Number and Op	erations in	Base Ten				Ratios and Proportion Relationsh	nal			ories
				Number an	d Operations	- Fractions	Number Sy	/stem			atego
	Operations and	Algebraic 1	Thinking				Expression	s and Equation	ons	Algebra	Conceptual Categories
									Functions	Functions	cept
	Geometry									Geometry	S
	Measurement a	nd Data					Statistics a	nd Probabilit	У	Statistics and Probability	

**Domains** are large groups of related standards. Each shaded row shows how domains at the earlier grades progress and lead to conceptual categories at the high school levels. The right side of the chart lists the five **conceptual categories** for high school. Selecting one conceptual category and moving left along the row shows the domains at the middle and elementary school levels from which this concept builds. Modeling, the sixth conceptual category, is incorporated throughout the other five high school categories.

Overall, the progressions of the standards begin and end in different grades, avoiding the re-teaching of concepts that should have been mastered. This allows for higher rigor overall, which is key to laying the foundation for high school mathematics standards and college/career preparedness.

For each of the grade-spans (K-2, 3-5, 6-8, and 9-12) an overview of the topics to be covered follows.

# **Overview of Mathematical Content Standards**

Kindergarten	Grade 1	Grade 2
Counting and Cardinality  Know number names and the count sequence.  Count to tell the number of objects.  Compare numbers.  Compare numbers.  Coperations and Algebraic Thinking  Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.  Identify and continue patterns.  Number and Operations in Base Ten  Work with numbers 11–19 to gain foundations for place value.  Measurement and Data  Describe and compare measurable attributes.  Classify objects and count the number of objects in categories.  Work with time and money.  Geometry  Identify and describe shapes.  Analyze, compare, create, and compose shapes.	<ul> <li>Counting and Cardinality</li> <li>Know ordinal names and counting flexibility.</li> <li>Count to tell the number of objects.</li> <li>Compare numbers.</li> <li>Operations and Algebraic Thinking</li> <li>Represent and solve problems involving addition and subtraction.</li> <li>Understand and apply properties of operations and the relationship between addition and subtraction.</li> <li>Add and subtract up to 20.</li> <li>Work with addition and subtraction equations.</li> <li>Identify and continue patterns.</li> <li>Number and Operations in Base Ten</li> <li>Extend the counting sequence.</li> <li>Understand place value</li> </ul>	Operations and Algebraic Thinking  Represent and solve problems involving addition and subtraction.  Add and subtract up to 20.  Work with equal groups of objects to gain foundations for multiplication.  Identify and continue patterns.  Number and Operations in Base Ten  Understand place value.  Use place value understanding and properties of operations to add and subtract.  Measurement and Data  Measure and estimate lengths in standard units.  Relate addition and subtraction to length.  Work with time and money.  Represent and interpret data.  Geometry  Reason with shapes and their attributes.

# Operations and Algebraic Thinking

Grade 3

- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide up to 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

### **Number and Operations in Base Ten**

 Use place value understanding and properties of operations to perform multi-digit arithmetic.

#### **Number and Operations—Fractions**

 Develop understanding of fractions as numbers.

#### **Measurement and Data**

- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- · Represent and interpret data.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

# Grade 4

#### **Operations and Algebraic Thinking**

- Use the four operations with whole numbers to solve problems.
- Gain familiarity with factors and multiples.
- Generate and analyze patterns.

#### **Number and Operations in Base Ten**

- Generalize place value understanding for multi-digit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

#### **Number and Operations—Fractions**

- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions, and compare decimal fractions.

#### Measurement and Data

- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit and involving time.
- · Represent and interpret data.
- Geometric measurement: understand concepts of angle and measure angles.

#### **Operations and Algebraic Thinking**

Grade 5

- Write and interpret numerical expressions.
- Analyze patterns and relationships.

#### **Number and Operations in Base Ten**

- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

#### **Number and Operations—Fractions**

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

#### **Measurement and Data**

- Convert like measurement units within a given measurement system and solve problems involving time.
- · Represent and interpret data.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

#### Geometry

 Graph points on the coordinate plane to solve real-world and mathematical problems.

Grade 3	Grade 4	Grade 5
<ul><li>Geometry</li><li>Reason with shapes and their attributes.</li></ul>	<ul> <li>Geometry</li> <li>Draw and identify lines and angles, and classify shapes by properties of their lines and angles.</li> </ul>	<ul> <li>Classify two-dimensional figures into categories based on their properties.</li> </ul>

Grade 6	Grade 7	Grade 8
<ul> <li>Ratios and Proportional Relationships</li> <li>Understand ratio concepts and use ratio reasoning to solve problems.</li> </ul>	<ul> <li>Ratios and Proportional Relationships</li> <li>Analyze proportional relationships and use them to solve real-world and mathematical problems.</li> </ul>	<ul> <li>The Number System</li> <li>Know that there are numbers that are not rational, and approximate them by rational numbers.</li> </ul>
The Number System	·	
<ul> <li>Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</li> <li>Compute fluently with multi-digit numbers and find common factors and multiples.</li> </ul>	<ul> <li>The Number System</li> <li>Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.</li> </ul>	<ul> <li>Expressions and Equations</li> <li>Work with radicals and integer exponents.</li> <li>Understand the connections between proportional relationships, lines, and linear equations.</li> </ul>
<ul> <li>Apply and extend previous understandings of numbers to the system of rational numbers.</li> </ul>	<ul> <li>Expressions and Equations</li> <li>Use properties of operations to generate equivalent expressions.</li> <li>Solve real-life and mathematical</li> </ul>	<ul> <li>Analyze and solve linear equations and pairs of simultaneous linear equations.</li> </ul>
e a contrar and experience		Geometry
<ul> <li>Expressions and Equations</li> <li>Apply and extend previous understandings of arithmetic to algebraic expressions.</li> <li>Reason about and solve one-variable equations and inequalities.</li> <li>Represent and analyze quantitative relationships between dependent and independent variables.</li> </ul>	problems using numerical and algebraic expressions and equations.  Geometry  Draw, construct and describe geometrical figures and describe the relationships between them.  Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	<ul> <li>Understand congruence and similarity using physical models, transparencies, or geometry software.</li> <li>Understand and apply the Pythagorear Theorem.</li> <li>Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.</li> <li>Statistics and Probability</li> </ul>
Geometry		Investigate patterns of association in
<ul> <li>Solve real-world and mathematical problems involving area, surface area, and volume.</li> </ul>	<ul> <li>Statistics and Probability</li> <li>Use random sampling to draw inferences about a population.</li> <li>Draw informal comparative inferences</li> </ul>	bivariate data.  Functions  • Define, evaluate, and compare

about two populations.

functions.

between quantities.

Use functions to model relationships

**Statistics and Probability** 

variability.

Develop understanding of statistical

Grade 6	Grade 7	Grade 8
Summarize and describe distributions.	<ul> <li>Investigate chance processes and develop, use, and evaluate probability models.</li> </ul>	

## **Overview of High School Content Standards**

Modeling	Number and Quantity	Algebra
Modeling  Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions.  Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Specific modeling standards appear throughout the high school standards indicated by an	<ul> <li>Number and Quantity</li> <li>The Real Number System</li> <li>Extend the properties of exponents to rational exponents.</li> <li>Use properties of rational and irrational numbers.</li> <li>Quantities*</li> <li>Reason quantitatively and use units to solve problems.</li> <li>The Complex Number System</li> <li>Perform arithmetic operations with complex numbers.</li> </ul>	<ul> <li>Seeing Structure in Expressions</li> <li>Interpret the structure of expressions.</li> <li>Write expressions in equivalent forms to solve problems.*</li> <li>Arithmetic with Polynomials and Rational Expressions</li> <li>Perform arithmetic operations on polynomials.</li> <li>Understand the relationship between zeros and factors of polynomials.</li> <li>Use polynomial identities to solve problems.</li> </ul>
asterisk (*).  If the asterisk appears on the heading for a group of standards, it should be understood to apply to all standards in that group. There are other individual standards under clusters, domains and conceptual categories that have connections to modeling.	<ul> <li>Represent complex numbers and their operations on the complex plane. +</li> <li>Use complex numbers in polynomial identities and equations.</li> <li>Vector and Matrix Quantities</li> <li>Represent and model with vector quantities. +</li> <li>Perform operations on vectors. +</li> <li>Perform operations on matrices and use matrices in applications. +</li> </ul>	<ul> <li>Rewrite rational expressions.</li> <li>Creating Equations and Inequalities*</li> <li>Create equations and inequalities that describe numbers or relationships.</li> <li>Reasoning with Equations and Inequalities</li> <li>Understand solving equations as a process of reasoning and explain the reasoning.</li> </ul>

Modeling	Number and Quantity	Algebra
Additionally, model with mathematics is a Standard for Mathematical Practice. This practice will be started in kindergarten.		<ul> <li>Solve equations and inequalities in one variable.</li> <li>Solve systems of equations.</li> <li>Represent and solve equations and inequalities graphically.</li> </ul>

<sup>\*</sup>Standards with connections to modeling. If asterisk appears on the category, domain, or cluster for a group of standards, it should be understood to apply to all standards in that group. There may be individual standards within clusters with connections to modeling.

<sup>+</sup> Standards include additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.

Functions	Geometry	Statistics and Probability*
<ul> <li>Interpreting Functions</li> <li>Understand the concept of a function and use function notation.</li> <li>Interpret functions that arise in applications in terms of the context.</li> <li>Analyze functions using different representations.</li> <li>Building Functions</li> <li>Build a function that models a</li> </ul>	<ul> <li>Congruence</li> <li>Experiment with transformations in the plane.</li> <li>Understand congruence in terms of rigid motions.</li> <li>Prove geometric theorems.</li> <li>Make geometric constructions.</li> <li>Similarity, Right Triangles, and Trigonometry</li> </ul>	<ul> <li>Interpreting Categorical and Quantitative Data</li> <li>Summarize, represent, and interpret data on a single count or measurement variable.</li> <li>Summarize, represent, and interpret data on two categorical and quantitative variables.</li> <li>Interpret linear models.</li> </ul>
<ul> <li>Build a function that models a relationship between two quantities.</li> <li>Build new functions from existing functions.</li> <li>Linear, Quadratic, and Exponential Models*</li> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> <li>Interpret expressions for functions in</li> </ul>	<ul> <li>Understand similarity in terms of similarity transformations.</li> <li>Prove theorems involving similarity.</li> <li>Define trigonometric ratios and solve problems involving right triangles.</li> <li>Apply trigonometry to general triangles. +</li> <li>Circles</li> <li>Understand and apply theorems</li> </ul>	<ul> <li>Making Inferences and Justifying Conclusions</li> <li>Understand and evaluate random processes underlying statistical experiments.</li> <li>Make inferences and justify conclusions from sample surveys, experiments, and observational studies.</li> </ul>
<ul> <li>terms of the situation they model.</li> <li>Trigonometric Functions</li> <li>Extend the domain of trigonometric functions using the unit circle.</li> <li>Model periodic phenomena with trigonometric functions.</li> <li>Prove and apply trigonometric identities.</li> </ul>	<ul> <li>about circles.</li> <li>Find arc lengths and areas of sectors of circles.</li> <li>Expressing Geometric Properties with Equations</li> <li>Translate between the geometric description and the equation for a conic section.</li> <li>Use coordinates to prove simple geometric theorems algebraically.</li> </ul>	<ul> <li>Conditional Probability and the Rules of Probability</li> <li>Understand independence and conditional probability and use them to interpret data.</li> <li>Use the rules of probability to compute probabilities of compound events in a uniform probability model.</li> <li>Using Probability to Make Decisions</li> <li>Calculate expected values and use them to solve problems. +</li> </ul>

Functions	Geometry	Statistics and Probability*
	Geometric Measurement and Dimensio  Explain volume formulas and use them to solve problems.  Visualize relationships between two dimensional and three-dimensional objects.  Modeling with Geometry  Apply geometric concepts in modeli situations.*	Use probability to evaluate outcomes of decisions. +

<sup>\*</sup>Standards with connections to modeling. If the asterisk appears on the category, domain, or cluster for a group of standards, it should be understood to apply to all standards in that group. There may be individual standards within clusters with connections to modeling.

<sup>+</sup> Standards include additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics.

# **Guide to Reading the Mathematical Content Standards**

There are eleven domains within the K-8 Standards. Students advancing through the grades are expected to meet each year's grade-specific standards, and retain or further develop skills and understandings mastered in preceding grades. An instructional focus is included before each grade to support the implementation of the content.

#### K-8 Mathematical Domains:

- 1. Counting and Cardinality CC
- 2. Operations and Algebraic Thinking OA
- 3. Number and Operations in Base Ten NBT
- 4. Measurement and Data MD
- 5. Number and Operations Fractions NF
- 6. Geometry G
- 7. Ratios and Proportional Relationships RP
- 8. The Number System NS
- 9. Expressions and Equations EE
- 10. Functions F
- 11. Statistics and Probability SP

**Domains** are intended to convey coherent groupings of content. All domains are underlined.

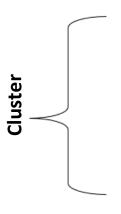
**Clusters** are groups of related standards. Cluster headings are bolded.

**Standards** define what students should understand and be able to do. Standards are numbered. Any standard followed by an (L) indicates the standard is to be locally assessed.

#### K- 8 Grade Level

#### Grade 4

#### **Domain** Number and Operations in Base Ten 4.NBT



#### Generalize place value understanding for multi-digit whole numbers.

- 4.NBT.1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that  $700 \div 70 = 10$  by applying concepts of place value and division.
- 4.NBT.2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on the value of the digits in each place, using >, =, and < symbols to record the results of comparisons.
- 4.NBT.3. Use place value understanding to round multi-digit whole numbers to any place using a variety of estimation methods; be able to describe, compare, and contrast solutions.

**Standards** 

{4.NBT.1, 4.NBT.2, & 4.NBT.3}

The high school standards specify the mathematics that all students should study in order to be career and college ready. They are organized into conceptual categories, which are intended to portray a coherent view of high school mathematics. A student's work with any set of standards crosses a number of traditional course boundaries. For example, the Functions Standards would apply to many courses such as Algebra I or Algebra II. It is a district decision how to design course offerings covering the mathematics standards. Districts can use the traditional approach of Algebra I, Geometry, and Algebra II or implement an integrated approach. There are various high school math pathways to be considered.

There are six conceptual categories for high school. Each conceptual category in the high school standards is prefaced with a narrative and an explanation of the implication of that category to a student's mastery of mathematics.

#### **High School Mathematical Conceptual Categories:**

- 1. Number and Quantity N
- 2. Algebra A
- 3. Functions F
- 4. Modeling M
- 5. Geometry G
- 6. Statistics and Probability P

- (+) Additional standards for advanced courses
- (\*) Standards with connection to modeling

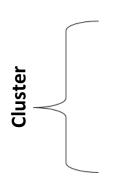
# High School Conceptual Category

### **Geometry Standards**

**Geometric Measurement and Dimension (G-GMD)** 

#### Domain

Explain volume formulas and use them to solve problems



#### Generalize place value understanding for multi-digit whole numbers.

- G-GMD.1. Explain how to find the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- G-GMD.2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
- G-GMD.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.\*

**Standards** { G-GMD.1, G-GMD.2, & G-GMD.3}

# **Standards for Mathematical Practice**

### **Alaska Standards for Mathematical Practice**

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

- 1. Make sense of problems and persevere in solving them
- 2. Reason abstractly and quantitatively
- 3. Construct viable arguments and critique the reasoning of others
- 4. Model with mathematics

- 5. Use appropriate tools strategically
- 6. Attend to precision
- 7. Look for and make use of structure
- 8. Look for and express regularity in repeated reasoning

Each Standard for Mathematical Practice listed below is followed by a set of grade-span descriptors. These descriptors of the Standards of Mathematical Practice are meant to help students, parents and educators to picture how these practices might be demonstrated by students. Within the grade span, students should apply the practices using specific grade-level content. Additionally, students at higher grade spans may revisit earlier grade-span proficiencies as the rigor of the content increases.

### **Connecting the Standards for Mathematical Practice and Mathematical Content**

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or

deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

### 1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

#### In grades K-2 mathematically proficient students will:

- focus on the problem and check for alternate methods
- check if the solution makes sense

### In grades 3-5 mathematically proficient students will:

- explain correspondences between equations, verbal descriptions, tables, and graphs
- draw diagrams of important features and relationships, graph data, and search for regularity or trends
- use concrete objects or pictures to help conceptualize and solve a problem
- understand the approaches of others to solving complex problems
- identify correspondences between different approaches

check if the solution makes sense

#### In grades 6-8 mathematically proficient students will:

- explain correspondences between a new problem and previous problems
- represent algebraic expressions numerically, graphically, concretely/with manipulatives, verbally/written
- explain connections between the multiple representations
- determine the question that needs to be answered
- analyze a problem and make a plan for solving it
- choose a reasonable strategy
- identify the knowns and unknowns in a problem
- use previous knowledge and skills to simplify and solve problems
- break a problem into manageable parts or simpler problems
- solve a problem in more than one way

#### In grades 9-12 mathematically proficient students will:

- make connections between a new problem and previous problems
- determine the question that needs to be answered
- choose a reasonable strategy
- identify the knowns and unknowns in a problem
- use previous knowledge and skills to simplify and solve problems
- break a problem into manageable parts or simpler problems
- represent algebraic expressions numerically, graphically, concretely/with manipulatives, verbally/written
- explain connections between the multiple representations
- solve a problem in more than one way
- explain the meaning of a problem and look for an entry point
- analyze a problem and make a plan for solving it
- · explain correspondence between differing approaches to identify regularity and trends
- check answer using a different method
- identify correspondence between different approaches
- monitor and evaluate progress and change course if necessary
- check the answers to problems using a different method and continually ask, "Does this make sense?"

## 2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

## In grades K-2 mathematically proficient students will:

- represent a situation symbolically and/or with manipulatives
- create a coherent representation of the problem
- use units of measurement consistently

## In grades 3-5 mathematically proficient students will:

- represent a situation symbolically
- create a coherent representation of the problem
- have the ability to show how problem has a realistic meaning
- reflect during the manipulation process in order to probe into the meanings for the symbols involved
- use units consistently

## In grades 6-8 mathematically proficient students will:

- represent a situation symbolically and carry out its operations
- create a coherent representation of the problem
- translate an algebraic problem to a real world context
- explain the relationship between the symbolic abstraction and the context of the problem
- compute using different properties
- consider the quantitative values, including units, for the numbers in a problem

#### In grades 9-12 mathematically proficient students will:

- decontextualize to abstract a given situation and represent it symbolically and manipulate the representing symbols.
- reflect during the manipulation process in order to probe into the meanings for the symbols involved
- create a coherent representation of the problem
- make sense of quantities and their relationships in problem situations
- attend to the meanings of quantities
- use flexibility with different properties of operations and objects
- translate an algebraic problem to a real world context
- explain the relationship between the symbolic abstraction and the context of the problem
- compute using different properties
- consider the quantitative values, including units, for the numbers in a problem

## 3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

## In grades K-2 mathematically proficient students will:

- construct arguments using concrete referents such as objects, drawings, diagrams, and actions
- justify conclusions, communicate conclusions
- listen to arguments and decide whether the arguments make sense

#### In grades 3-5 mathematically proficient students will:

- construct arguments using concrete referents such as objects, drawings, diagrams, and actions
- justify conclusions, communicate conclusions, listen and respond to arguments, decide whether the argument makes sense, and ask questions to clarify the argument
- reason inductively about data, making plausible arguments that take into account the context from which the data arose

#### In grades 6-8 mathematically proficient students will:

- construct arguments using both concrete and abstract explanations
- justify conclusions, communicate conclusions, and respond to the arguments
- listen to arguments, critique their viability, and ask questions to clarify the argument
- compare effectiveness of two arguments by identifying and explaining both logical and/or flawed reasoning
- recognize general mathematical truths and use statements to justify the conjectures
- identify special cases or counter-examples that don't follow the mathematical rules
- infer meaning from data and make arguments using its context

## In grades 9-12 mathematically proficient students will:

- construct arguments using both concrete and abstract explanations
- justify conclusions in a variety of ways, communicate the methodology, and respond to the arguments
- reason inductively about data and make plausible arguments that take into account the context from which the data arose
- understand and use stated assumptions, definitions, and previously established results in constructing arguments
- make conjectures and build a logical progression of statements to explore the truth of the conjectures
- analyze situations by breaking them into cases and recognize and use counter-examples
- recognize general mathematical truths and statements to justify the conjectures
- identify special cases or counter-examples that don't follow the mathematical rules
- infer meaning from data and make arguments using its context
- compare effectiveness of two arguments by identifying and explaining both logical and/or flawed reasoning

## 4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## In grades K-2 mathematically proficient students will:

- apply mathematics to solve problems in everyday life
- identify important quantities in a practical situation and model the situation with manipulatives or pictures
- interpret mathematical results in the context of the situation and reflect on whether the results make sense

## In grades 3-5 mathematically proficient students will:

- apply mathematics to solve problems arising in everyday life
- identify important quantities in a practical situation and model the situation using such tools as manipulatives, diagrams, two-way tables, graphs or pictures
- interpret mathematical results in the context of the situation and reflect on whether the results make sense
- apply mathematical knowledge, make assumptions and approximations to simplify a complicated situation

## In grades 6-8 mathematically proficient students will:

- apply mathematics to solve problems arising in everyday life and society
- identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, and formulas
- interpret their mathematical results in the context of the situation and reflect on whether the results make sense
- make assumptions and approximations to simplify a situation, realizing the final solution will need to be revised
- analyze quantitative relationships to draw conclusions
- reflect on whether their results make sense

• improve the model if it has not served its purpose

## In grades 9-12 mathematically proficient students will:

- apply mathematics to solve problems in everyday life, society, and workplace
- identify important quantities in a practical situation and map the relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas
- consistently interpret mathematical results in the context of the situation and reflect on whether the results make sense
- apply knowledge, making assumptions and approximations to simplify a complicated situation, realizing that these may need
  revision later
- make assumptions and approximations to simplify a situation, realizing the final solution will need to be revised
- identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, and formulas
- analyze quantitative relationships to draw conclusions
- improve the model if it has not served its purpose

## 5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## In grades K-2 mathematically proficient students will:

 select the available tools (such as pencil and paper, manipulatives, rulers, and available technology) when solving a mathematical problem

- be familiar with tools appropriate for the grade level to make sound decisions about when each of these tools might be helpful
- identify relevant external mathematical resources and use them to pose or solve problems
- use technological tools to explore and deepen their understanding of concepts

## In grades 3-5 mathematically proficient students will:

- select the available tools (such as pencil and paper, manipulatives, rulers, calculators, a spreadsheet, and available technology) when solving a mathematical problem
- be familiar with tools appropriate for their grade level to make sound decisions about when each of these tools might be helpful
- identify relevant external mathematical resources and use them to pose or solve problems
- use technological tools to explore and deepen their understanding of concepts
- detect possible errors by strategically using estimation and other mathematical knowledge
- know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data

## In grades 6-8 mathematically proficient students will:

- select and use tools appropriate to the task: pencil and paper, protractor, visual and physical fraction models, algebra tiles, geometric models, calculator, spreadsheet, and interactive geometry software.
- use estimation and other mathematical knowledge to confirm the accuracy of their problem solving
- identify relevant external and digital mathematical resources and use them to pose or solve problems
- represent and compare possibilities visually with technology when solving a problem
- explore and deepen their understanding of concepts through the use of technological tools

## In grades 9-12 mathematically proficient students will:

- select and accurately use appropriate, available tools (such as pencil and paper, concrete or virtual manipulatives such as geoboards and algebra tiles, graphing and simpler calculators, a spreadsheet, and available technology) when solving a mathematical problem
- identify relevant external and digital mathematical resources and use the resources to pose or solve problems
- detect possible errors by strategically using estimation and other mathematical knowledge
- use technology to visualize the results of varying assumptions, exploring consequences, comparing predictions with data, and deepening understanding of concepts

## 6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

### In grades K-2 mathematically proficient students will:

- give thoughtful explanations to each other
- use clear definitions and reasoning in discussion with others
- state the meaning of symbols they choose, including using the equal sign consistently and appropriately

## In grades 3-5 mathematically proficient students will:

- give carefully formulated explanations to each other
- use clear definitions and reasoning in discussion with others
- state the meaning of symbols, including using the equal sign consistently and appropriately
- specify units of measure, and label axes to clarify the correspondence with quantities in a problem
- calculate accurately and efficiently
- express numerical answers with a degree of precision appropriate for the problem context

## In grades 6-8 mathematically proficient students will:

- use clear definitions in explanations
- understand and use specific symbols accurately and consistently: equality, inequality, ratios, parenthesis for multiplication and division, absolute value, square root
- specify units of measure, and label axes to clarify the correspondence with quantities in a problem
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context

## In grades 9-12 mathematically proficient students will:

communicate precisely to others

- use clear definitions in explanations
- use symbols consistently and appropriately
- specify units of measure, and label axes to clarify the correspondence with quantities in a problem
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context
- examine claims and make explicit use of definitions

## 7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression

 $x^2 + 9x + 14$ , older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see

 $5-3(x-y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

## In all grade levels mathematically proficient students will:

- discern a pattern or structure
- understand complex structures as single objects or as being composed of several objects
- check if the answer is reasonable

## 8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y - 2)/(x - 1) = 3. Noticing the regularity in the way terms cancel when expanding (x - 1)(x + 1),  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general

formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## In all grade levels mathematically proficient students will:

- identify if calculations or processes are repeated
- use alternative and traditional methods to solve problems
- evaluate the reasonableness of their intermediate results, while attending to the details

# **K-8 Mathematical Content Standards**

# **Instructional Focus: Kindergarten through Second Grade**

## Kindergarten

In Kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

(1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 - 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

#### Grade 1

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

(1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and

#### Grade 2

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

- (1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).
- (2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their

## Kindergarten

(2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

#### Grade 1

use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

- (2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.
- (3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.<sup>1</sup>

### Grade 2

understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

- (3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.
- (4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Kindergarten	Grade 1	Grade 2
	(4) Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry	
	<sup>1</sup> Students should apply the principle of transitivity of measurement to make	
	indirect comparisons, but they need not use this technical term.	

# **Alaska Mathematics Standards Grades K-2**

Grade K	Grade 1	Grade 2
Counting and Cardinality K.CC	Counting and Cardinality, 1.CC	
Know number names and the count sequence.	Know ordinal names and counting flexibility.	
K.CC.1. Count to 100 by ones and by tens.  K.CC.2. Count forward beginning from a given number within the known sequence.  K.CC.3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0 - 20 (with 0 representing a count of no objects).  Count to tell the number of objects.  K.CC.4. Understand the relationship between numbers and quantities; connect counting to cardinality.	1.CC.1. Skip count by 2s and 5s.  1.CC.2. Use ordinal numbers correctly when identifying object position (e.g., first, second, third, etc.).  1.CC.3. Order numbers from 1-100. Demonstrate ability in counting forward and backward.  Count to tell the number of objects.  1.CC.4. Count a large quantity of objects by grouping into 10s and counting by 10s and 1s to find the quantity.	
a. When counting objects, say the number names in standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.		

Grade K	Grade 1	Grade 2
K.CC.5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.  Compare numbers.  K.CC.6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group (e.g., by using matching, counting, or estimating strategies).  K.CC.7. Compare and order two numbers between 1 and 10 presented as written numerals.	Compare numbers.  1.CC.5. Use the symbols for greater than, less than or equal to when comparing two numbers or groups of objects.  1.CC.6. Estimate how many and how much in a given set to 20 and then verify estimate by counting.	
Operations and Algebraic Thinking K.OA  Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.  K.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps) acting out situations, verbal explanations, expressions, or equations.	Represent and solve problems involving addition and subtraction.  1.OA.1. Use addition and subtraction strategies to solve word problems (using numbers up to 20), involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, using a number line (e.g., by using objects, drawings and equations). Record and explain using equation symbols and a symbol for the unknown number to represent the problem.	Represent and solve problems involving addition and subtraction.  2.OA.1. Use addition and subtraction strategies to estimate, then solve one- and two-step word problems (using numbers up to 100) involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions (e.g., by using objects, drawings and equations). Record and explain using equation symbols and a symbol for the unknown number to represent the problem.

#### Grade K Grade 1 Grade 2 K.OA.2. Add or subtract whole numbers 1.OA.2. Solve word problems that call for Add and subtract using numbers up to addition of three whole numbers whose to 10 (e.g., by using objects or drawings to solve word problems). sum is less than or equal to 20 (e.g., by 2.OA.2. Fluently add and subtract using using objects, drawings and equations). K.OA.3. Decompose numbers less than or numbers up to 20 using mental strategies. Record and explain using equation equal to 10 into pairs in more than one symbols and a symbol for the unknown Know from memory all sums of two oneway (e.g., by using objects or drawings, digit numbers. number to represent the problem. and record each decomposition by a Work with equal groups of objects to drawing or equation). For example, 5 = 2Understand and apply properties of gain foundations for multiplication. + 3 and 5 = 4 + 1.operations and the relationship between addition and subtraction. 2.OA.3. Determine whether a group of K.OA.4. For any number from 1 - 4, find objects (up to 20) is odd or even (e.g., by the number that makes 5 when added to 1.OA.3. Apply properties of operations as pairing objects and comparing, counting strategies to add and subtract. (Students the given number and, for any number by 2s). Model an even number as two from 1 - 9, find the number that makes 10 need not know the name of the property.) equal groups of objects and then write an when added to the given number (e.g., by For example: If 8 + 3 = 11 is known, then equation as a sum of two equal addends. using objects, drawings or 10 frames) and 3 + 8 = 11 is also known (Commutative record the answer with a drawing or property of addition). To add 2 + 6 + 4, 2.OA.4. Use addition to find the total the second two numbers can be added to equation. number of objects arranged in rectangular *make a ten. so* 2 + 6 + 4 = 2 + 10 = 12arrays with up to 5 rows and up to 5 K.OA.5. Fluently add and subtract (Associative property of addition). columns. Write an equation to express the numbers up to 5. Demonstrate that when adding zero to any total as repeated addition (e.g., array of 4 number, the quantity does not change Identify and continue patterns. by 5 would be (Identity property of addition). K.OA.6. Recognize, identify and continue 5+5+5+5=20). simple patterns of color, shape, and size. 1.OA.4. Understand subtraction as an Identify and continue patterns. unknown-addend problem. For example, Number and Operations in Base Ten 2.OA.5. Identify, continue and label subtract 10 - 8 by finding the number that K.NBT number patterns (e.g., aabb, abab). makes 10 when added to 8. Describe a rule that determines and Add and subtract using numbers up to Work with numbers 11-19 to gain continues a sequence or pattern. 20. foundations for place value. Number and Operations in Base Ten 1.OA.5. Relate counting to addition and K.NBT.1. Compose and decompose 2.NBT subtraction (e.g., by counting on 2 to add numbers from 11 to 19 into ten ones and

Understand place value.

some further ones (e.g., by using objects

2).

Grade K	Grade 1	Grade 2
or drawings) and record each composition and decomposition by a drawing or equation (e.g., $18 = 10 + 8$ ); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight or nine ones.	<ul> <li>1.OA.6. Add and subtract using numbers up to 20, demonstrating fluency for addition and subtraction up to 10. Use strategies such as</li> <li>counting on</li> <li>making ten (8+6=8+2+4=10+4=14)</li> <li>decomposing a number leading to a ten (13-4=13-3-1=10-1=9)</li> <li>using the relationship between addition and subtraction, such as fact families, (8+4=12 and 12-8=4)</li> <li>creating equivalent but easier or known sums (e.g., adding 6+7 by creating the known equivalent 6+6+1=12+1=13).</li> </ul>	2.NBT.1. Model and identify place value positions of three digit numbers. Include: a. 100 can be thought of as a bundle of ten tenscalled a "hundred". b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).  2.NBT.2. Count up to 1000, skip-count by 5s, 10s and 100s.  2.NBT.3. Read, write, order up to 1000 using base-ten numerals, number names and expanded form. 2.NBT.4. Compare two three-digit numbers based on the meanings of the hundreds, tens and ones digits, using >, =, < symbols to record the results.
	Work with addition and subtraction equations.	
	1.OA.7. Understand the meaning of the equal sign (e.g., read equal sign as "same as") and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? $6 = 6$ , $7 = 8 - 1$ , $5 + 2 = 2 + 5$ , $4 + 1 = 5 + 2$ .	
	1.OA.8. Determine the unknown whole number in an addition or subtraction equation. For example, determine the unknown number that makes the equation	

Grade K	Grade 1	Grade 2
	true in each of the equations $8 + ? = 11$ , $6 + 6 = ?$ , $5 = ? - 3$	
	Identify and continue patterns. 1.OA.9. Identify, continue and label patterns (e.g., aabb, abab). Create patterns using number, shape, size, rhythm or color.	
	Number and Operations in Base Ten 1.NBT	
	Extend the counting sequence. 1.NBT.1. Count to 120. In this range, read, write and order numerals and represent a number of objects with a written numeral.	
	Understand place value.  1.NBT.2. Model and identify place value positions of two digit numbers. Include: a. 10 can be thought of as a bundle of ten ones, called a "ten". b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90, refer to one, two, three, four, five, six, seven, eight or nine tens (and 0 ones).	
	1.NBT.3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, <.	

Grade K	Grade 1	Grade 2
	Use place value understanding and properties of operations to add and subtract.	Use place value understanding and properties of operations to add and subtract.
	1.NBT.4. Add using numbers up to 100 including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10.  Use:  • concrete models or drawings and strategies based on place value • properties of operations • and/or relationship between addition and subtraction.  Relate the strategy to a written method and explain the reasoning used.  Demonstrate in adding two-digit numbers, tens and tens are added, ones and ones are added and sometimes it is necessary to compose a ten from ten ones.  1.NBT.5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.  1.NBT.6. Subtract multiples of 10 up to 100. Use:  • concrete models or drawings • strategies based on place value • properties of operations • and/or the relationship between addition and subtraction.  Relate the strategy to a written method and explain the reasoning used.	<ul> <li>2.NBT.5. Fluently add and subtract using numbers up to 100.</li> <li>Use: <ul> <li>strategies based on place value</li> <li>properties of operations</li> <li>and/or the relationship between addition and subtraction.</li> </ul> </li> <li>2.NBT.6. Add up to four two-digit numbers using strategies based on place value and properties of operations.</li> <li>2.NBT.7. Add and subtract using numbers up to 1000.</li> <li>Use: <ul> <li>concrete models or drawings and strategies based on place value</li> <li>properties of operations</li> <li>and/or relationship between addition and subtraction.</li> </ul> </li> <li>Relate the strategy to a written method and explain the reasoning used.</li> <li>Demonstrate in adding or subtracting three-digit numbers, hundreds and hundreds are added or subtracted, tens and tens are added or subtracted, ones and ones are added or subtracted and sometimes it is necessary to compose a ten from ten ones or a hundred from ten tens.</li> </ul>

Grade K	Grade 1	Grade 2
		2.NBT.8. Mentally add 10 or 100 to a given number 100-900 and mentally subtract 10 or 100 from a given number.
		2.NBT.9. Explain or illustrate the processes of addition or subtraction and their relationship using place value and the properties of operations.
Measurement and Data K.MD	Measurement and Data 1.MD	Measurement and Data 2.MD
Describe and compare measurable attributes.	Measure lengths indirectly and by iterating length units.	Measure and estimate lengths in standard units.
K.MD.1. Describe measurable attributes of objects (e.g., length or weight). Match measuring tools to attribute (e.g., ruler to length). Describe several measurable attributes of a single object.	1.MD.1. Measure and compare three objects using standard or non-standard units.      1.MD.2. Express the length of an object as	2.MD.1. Measure the length of an object by selecting and using standard tools such as rulers, yardsticks, meter sticks, and measuring tapes.
K.MD.2. Make comparisons between two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. <i>For example, directly</i>	a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.	2.MD.2. Measure the length of an object twice using different length units for the two measurements. Describe how the two measurements relate to the size of the unit chosen.
compare the heights of two children and describe one child as taller/shorter.	Work with time and money.	2.MD.3. Estimate, measure and draw lengths using whole units of inches, feet, yards, centimeters and meters.
Classify objects and count the number of objects in each category.	1.MD.3. Tell and write time in half hours using both analog and digital clocks.	2.MD.4. Measure to compare lengths of two objects, expressing the difference in
K.MD.3. Classify objects into given categories (attributes). Count the number	1.MD.4. Read a calendar distinguishing yesterday, today and tomorrow. Read and	terms of a standard length unit.
of objects in each category (limit category counts to be less than or equal to 10).	write a date.	Relate addition and subtraction to length.

Grade K	Grade 1	Grade 2
Work with time and money.  K.MD.4. Name in sequence the days of the week.  K.MD.5. Tell time to the hour using both analog and digital clocks.  K.MD.6. Identify coins by name.	1.MD.5. Recognize and read money symbols including \$ and \$\psi.\$  1.MD.6. Identify values of coins (e.g., nickel = 5 cents, quarter = 25 cents). Identify equivalent values of coins up to \$1 (e.g., 5 pennies = 1 nickel, 5 nickels = 1 quarter).  Represent and interpret data.  1.MD.7. Organize, represent and interpret data with up to three categories. Ask and answer comparison and quantity questions about the data.	2.MD.5. Solve addition and subtraction word problems using numbers up to 100 involving length that are given in the same units (e.g., by using drawings of rulers). Write an equation with a symbol for the unknown to represent the problem.  2.MD.6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1,2,, and represent whole-number sums and differences within 100 on a number line diagram.  Work with time and money.  2.MD.7. Tell and write time to the nearest five minutes using a.m. and p.m. from analog and digital clocks.  2.MD.8. Solve word problems involving dollar bills and coins using the \$ and ¢ symbols appropriately.  Represent and interpret data.  2.MD.9. Collect, record, interpret, represent, and describe data in a table, graph or line plot.  2.MD.10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart and compare problems using information presented in a bar graph.

Grade K	Grade 1	Grade 2
Geometry K.G (shapes include squares, circles, triangles,	Geometry 1.G	Geometry 2.G
rectangles, hexagons, cubes, cones, cylinders, and spheres)	Reason with shapes and their attributes.	Reason with shapes and their attributes.
Identify and describe shapes. K.G.1. Describe objects in the environment using names of shapes and describe their relative positions (e.g., above, below, beside, in front of, behind, next to).  K.G.2. Name shapes regardless of their orientation or overall size.  K.G.3. Identify shapes as two-dimensional (flat) or three-dimensional (solid).  Analyze, compare, create, and compose shapes.  K.G.4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices), and other attributes (e.g., having sides of equal lengths).  K.G.5. Build shapes (e.g., using sticks and clay) and draw shapes.  K.G.6. Put together two-dimensional shapes to form larger shapes (e.g., join two triangles with full sides touching to	1.G.1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes. Identify shapes that have non-defining attributes (e.g., color, orientation, overall size). Build and draw shapes given specified attributes.  1.G.2. Compose (put together) two-dimensional or three-dimensional shapes to create a larger, composite shape, and compose new shapes from the composite shape.  1.G.3. Partition circles and rectangles into two and four equal shares. Describe the shares using the words, <i>halves</i> , <i>fourths</i> , and <i>quarters</i> and phrases <i>half of</i> , <i>fourth of</i> and <i>quarter of</i> . Describe the whole as two of or four of the shares. Understand for these examples that decomposing (break apart) into more equal shares creates smaller shares.	2.G.1. Identify and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces compared visually, not by measuring. Identify triangles, quadrilaterals, pentagons, hexagons and cubes.  2.G.2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.  2.G.3. Partition circles and rectangles into shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

## **Instructional Focus: Third Grade through Fifth Grade**

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

Grade 3

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equalsized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

Grade 4

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or

Grade 5

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of

#### Grade 3

- (2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.
- (3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or

#### Grade 4

mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fraction to multiply a fraction by a whole number.

#### Grade 5

dividing unit fractions by whole numbers and whole numbers by unit fractions.)

- (2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.
- (3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They

Grade 3	Grade 4	Grade 5
into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.  (4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.	(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.	understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

## **Alaska Mathematics Standards Grades 3-5**

Grade 3	Grade 4	Grade 5
Operations and Algebraic Thinking 3.OA	Operations and Algebraic Thinking 4.OA	Operations and Algebraic Thinking 5.OA
Represent and solve problems involving multiplication and division.	Use the four operations with whole numbers to solve problems.	Write and interpret numerical expressions.
3.OA.1. Interpret products of whole numbers (e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each). For example, show objects in rectangular arrays or describe a context	4.OA.1. Interpret a multiplication equation as a comparison e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 groups of 7 and 7 groups of 5 (Commutative property). Represent verbal statements of	5.OA.1. Use parentheses to construct numerical expressions, and evaluate numerical expressions with these symbols.
in which a total number of objects can be expressed as $5 \times 7$ .	multiplicative comparisons as multiplication equations.	5.OA.2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without
3.OA.2. Interpret whole-number quotients of whole numbers (e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of	4.OA.2. Multiply or divide to solve word problems involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem or missing numbers in an array). Distinguish	evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$ . Recognizing that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$ , without having to calculate the indicated sum or product.
8 objects each). For example, deconstruct rectangular arrays or describe a context	multiplicative comparison from additive comparison.	Analyze patterns and relationships.
<ul> <li>in which a number of shares or a number of groups can be expressed as 56 ÷ 8.</li> <li>3.OA.3. Use multiplication and division</li> </ul>	4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four	5.OA.3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding
numbers up to 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).	operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation	terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the
3.OA.4. Determine the unknown whole	strategies including rounding.	starting number 0, generate terms in the resulting sequences, and observe that the

terms in one sequence are twice the

number in a multiplication or division

Grade 3	Grade 4	Grade 5
equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$ , $5 = ? \div 3$ , $6 \times 6 = ?$	Gain familiarity with factors and multiples. 4.OA.4.	corresponding terms in the other sequence. Explain informally why this is so.
<ul> <li>Understand properties of multiplication and the relationship between multiplication and division.</li> <li>3.OA.5. Make, test, support, draw conclusions and justify conjectures about properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.)</li> <li>Commutative property of multiplication: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known.</li> <li>Associative property of multiplication: 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30.</li> <li>Distributive property: Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56.</li> <li>Inverse property (relationship) of multiplication and division.</li> </ul>	<ul> <li>Find all factor pairs for a whole number in the range 1–100.</li> <li>Explain the correlation/differences between multiples and factors.</li> <li>Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number.</li> <li>Determine whether a given whole number in the range 1–100 is prime or composite.</li> <li>Generate and analyze patterns.</li> <li>4.OA.5. Generate a number, shape pattern, table, t-chart, or input/output function that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. Be able to express the pattern in algebraic terms. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</li> </ul>	
3.OA.6. Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.	4.OA.6. Extend patterns that use addition, subtraction, multiplication, division or symbols, up to 10 terms, represented by	

Grade 3	Grade 4	Grade 5
Multiply and divide up to 100.	models (function machines), tables, sequences, or in problem situations. (L)	
3.OA.7. Fluently multiply and divide numbers up to 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$ , one knows $40 \div 5 = 8$ ) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.		
Solve problems involving the four operations, and identify and explain patterns in arithmetic.		
3.OA.8. Solve and create two-step word problems using any of the four operations. Represent these problems using equations with a symbol (box, circle, question mark) standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.		
3.OA.9. Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.		

Grade 3	Grade 4	Grade 5
Number and Operations in Base Ten 3.NBT  Use place value understanding and properties of operations to perform multi-digit arithmetic.  3.NBT.1. Use place value understanding to round whole numbers to the nearest 10 or 100.  3.NBT.2. Use strategies and/or algorithms to fluently add and subtract with numbers up to 1000, demonstrating understanding of place value, properties of operations, and/or the relationship between addition and subtraction.	Number and Operations in Base Ten 4.NBT  Generalize place value understanding for multi-digit whole numbers.  4.NBT.1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.  4.NBT.2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on the value of the digits in each place, using >, =, and <	Number and Operations in Base Ten 5.NBT  Understand the place value system.  5.NBT.1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.  5.NBT.2. Explain and extend the patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain and extend the patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents
3.NBT.3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 x 80, 10 x 60) using strategies based on place value and properties of operations.	symbols to record the results of comparisons.  4.NBT.3. Use place value understanding to round multi-digit whole numbers to any place using a variety of estimation methods; be able to describe, compare, and contrast solutions.  Use place value understanding and properties of operations to perform multi-digit arithmetic.	to denote powers of 10.  5.NBT.3. Read, write, and compare decimals to thousandths.  a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form [e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 (1/10) + 9 (1/100) + 2 (1/1000)].  b. Compare two decimals to thousandths place based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
	4.NBT.4. Fluently add and subtract multidigit whole numbers using any algorithm. Verify the reasonableness of the results.	record the results of comparisons.

Grade 3	Grade 4	Grade 5
	4.NBT.5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.  4.NBT.6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	5.NBT.4. Use place values understanding to round decimals to any place.  Perform operations with multi-digit whole numbers and with decimals to hundredths.  5.NBT.5. Fluently multiply multi-digit whole numbers using a standard algorithm.  5.NBT.6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, number lines, real life situations, and/or area models.  5.NBT.7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between the operations. Relate the strategy to a written method and explain their reasoning in getting their answers.

#### Grade 3 **Grade 4** Grade 5 Number and Operations—Fractions 3.NF Number and Operations—Fractions 4.NF Number and Operations—Fractions 5.NF (limited in this grade to fractions with (limited in this grade to fractions with Use equivalent fractions as a strategy to denominators 2, 3, 4, 6, and 8) denominators 2, 3, 4, 5, 6, 8, 10, 12, and add and subtract fractions. 100) **Develop understanding of fractions as** 5.NF.1. Add and subtract fractions with numbers. **Extend understanding of fraction** unlike denominators (including mixed equivalence and ordering. 3.NF.1. Understand a fraction 1/b (e.g., numbers) by replacing given fractions with equivalent fractions in such a way as 1/4) as the quantity formed by 1 part when 4.NF.1. Explain why a fraction a/b is a whole is partitioned into b (e.g., 4) equivalent to a fraction $(n \times a)/(n \times b)$ by to produce an equivalent sum or equal parts; understand a fraction *a/b* using visual fraction models, with difference of fractions with like (e.g., 2/4) as the quantity formed by a attention to how the number and size of denominators. For example, 2/3 + 5/4 =(e.g., 2) parts of size 1/b. (e.g., 1/4) the parts differ even though the two 8/12 + 15/12 = 23/12. (In general, a/b + 15/12 = 23/12) fractions themselves are the same size. c/d = (ad + bc)/bd.) 3.NF.2. Understand a fraction as a number Use this principle to recognize and on the number line; represent fractions on 5.NF.2. Solve word problems involving generate equivalent fractions. addition and subtraction of fractions a number line diagram. a. Represent a fraction 1/b (e.g., 1/4) on a 4.NF.2. Compare two fractions with referring to the same whole, including number line diagram by defining the cases of unlike denominators (e.g., by different numerators and different interval from 0 to 1 as the whole and denominators (e.g., by creating common using visual fraction models or equations partitioning it into b (e.g., 4) equal parts. to represent the problem). Use benchmark denominators or numerators, or by comparing to a benchmark fraction such fractions and number sense of fractions to Recognize that each part has size 1/b (e.g., 1/4) and that the endpoint of the part as 1/2). Recognize that comparisons are estimate mentally and check the based at 0 locates the number 1/b (e.g., valid only when the two fractions refer to reasonableness of answers. For example, 1/4) on the number line. the same whole. Record the results of recognize an incorrect result 2/5 + 1/2 =b. Represent a fraction a/b (e.g., 2/8) on a comparisons with symbols >, =, or <, and 3/7, by observing that 3/7 < 1/2.

justify the conclusions (e.g., by using a

visual fraction model).

number line.

number line diagram or ruler by marking

Recognize that the resulting interval has size a/b (e.g., 2/8) and that its endpoint locates the number a/b (e.g., 2/8) on the

off a lengths 1/b (e.g., 1/8) from 0.

Grade 3	Grade 4	Grade 5
3.NF.3. Explain equivalence of fractions in	Build fractions from unit fractions by	
special cases, and compare fractions by	applying and extending previous	
reasoning about their size.	understandings of operations on whole	
a. Understand two fractions as equivalent	numbers.	
if they are the same size (modeled) or the		
same point on a number line.	4.NF.3. Understand a fraction $a/b$ with $a >$	
b. Recognize and generate simple	1 as a sum of fractions $1/b$ .	
equivalent fractions (e.g., $1/2 = 2/4$ , $4/6 =$	a. Understand addition and subtraction of	
2/3). Explain why the fractions are	fractions as joining and separating parts	
equivalent (e.g., by using a visual fraction	referring to the same whole.	
model).	b. Decompose a fraction into a sum of	
c. Express and model whole numbers as	fractions with the same denominator in	
fractions, and recognize and construct	more than one way, recording each	
fractions that are equivalent to whole	decomposition by an equation. Justify	
numbers. For example: Express 3 in the	decompositions (e.g., by using a visual	
form $3 = 3/1$ ; recognize that $6/1 = 6$ ;	fraction model). For example:	
locate 4/4 and 1 at the same point of a	3/8 = 1/8 + 1/8 + 1/8; $3/8 = 1/8 + 2/8$ ; 2	
number line diagram.	1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.	
d. Compare two fractions with the same	c. Add and subtract mixed numbers with	
numerator or the same denominator by	like denominators (e.g., by replacing each	
reasoning about their size. Recognize that	mixed number with an equivalent fraction,	
comparisons are valid only when the two	and/or by using properties of operations	
fractions refer to the same whole. Record	and the relationship between addition and	
the results of comparisons with the	subtraction).	
symbols $>$ , =, or $<$ , and justify the	d. Solve word problems involving addition	
conclusions (e.g., by using a visual fraction	and subtraction of fractions referring to the	
model).	same whole and having like denominators	
	(e.g., by using visual fraction models and	
	equations to represent the problem).	

4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction $a/b$ as a multiple of $1/b$ . For example, use $a$ visual fraction $model$ to represent $5/4$ as the product $5 \times (1/4)$ , recording the conclusion by the equation  b. Understand a multiple of $a/b$ as a multiple of $1/b$ , and use this understanding to multiply a fraction by a whole number. For example, use $a$ visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$ , recognizing this product as $6/5$ . (In general, $n \times (a/b) = (n \times a)/b$ .)  c. Solve word problems involving multiplication of a fraction by a whole number (e.g., by using visual fraction models and equations to represent the problem). Check for the reasonableness of the answer. For example, if each person at a party will eat $3/8$ of a pound of roast beef, and there will be $5$ people at the problem.	Grade 3	Grade 4	Grade 5
be needed? Between what two whole	Grade 3	4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.  a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).  b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)  c. Solve word problems involving multiplication of a fraction by a whole number (e.g., by using visual fraction models and equations to represent the problem). Check for the reasonableness of the answer. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will	5.NF.5 Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying $a/b$ by 1. (Division of a fraction by a fraction is not a requirement at this grade.)  5.NF.6. Solve real world problems involving multiplication of fractions and mixed numbers (e.g., by using visual

Grade 3	Grade 4	Grade 5
	Understand decimal notation for fractions, and compare decimal fractions.  4.NF.5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.  4.NF.6. Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.  4.NF.7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual model).	5.NF.7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.  a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3. b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4. c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions (e.g., by using visual fraction models and equations to represent the problem). For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

Grade 3	Grade 4	Grade 5
Measurement and Data 3.MD	Measurement and Data 4.MD	Measurement and Data 5.MD
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.  3.MD.1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes or hours (e.g., by representing the problem on a number line diagram or clock).  3.MD.2. Estimate and measure liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm³ and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve and create one-step word problems involving masses or volumes that are given in the same units (e.g., by using drawings, such as a beaker with a measurement scale, to represent the problem). (Excludes multiplicative comparison problems [problems involving notions of "times as much."])	Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit, and involving time.  4.MD.1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4-ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36).  4.MD.2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	Convert like measurement units within a given measurement system and solve problems involving time.  5.MD.1. Identify, estimate measure, and convert equivalent measures within systems English length (inches, feet, yards, miles) weight (ounces, pounds, tons) volume (fluid ounces, cups, pints, quarts, gallons) temperature (Fahrenheit) Metric length (millimeters, centimeters, meters, kilometers) volume (milliliters, liters), temperature (Celsius), (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems using appropriate tools.  5.MD.2. Solve real-world problems involving elapsed time between world time zones. (L)

Grade 3	Grade 4	Grade 5
3.MD.3. Select an appropriate unit of English, metric, or non-standard measurement to estimate the length, time, weight, or temperature (L)  Represent and interpret data. 3.MD.4. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.	4.MD.3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.  4.MD.4. Solve real-world problems involving elapsed time between U.S. time zones (including Alaska Standard time). (L)  Represent and interpret data.	Represent and interpret data.  5.MD.3. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.  5.MD.4. Explain the classification of data from real-world problems shown in graphical representations including the use of terms mean and median with a given set of data. (L)
3.MD.5. Measure and record lengths using rulers marked with halves and fourths of an inch. Make a line plot with the data, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.  3.MD.6. Explain the classification of data from real-world problems shown in graphical representations. Use the terms minimum and maximum. (L)	<ul> <li>4.MD.5. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.</li> <li>4.MD.6. Explain the classification of data from real-world problems shown in graphical representations including the use of terms range and mode with a given set of data. (L)</li> </ul>	

# Grade 3 Grade 4 Grade 5 Geometric measurement: understand concepts of area and relate area to concepts of angle and measure angles. Geometric measurement: understand concepts of volume concepts of angle and measure angles.

3.MD.7. Recognize area as an attribute of plane figures and understand concepts of area measurement.

multiplication and to addition.

- a. A square with side length 1 unit is said to have "one square unit" and can be used to measure area.
- b. Demonstrate that a plane figure which can be covered without gaps or overlaps by n (e.g., 6) unit squares is said to have an area of n (e.g., 6) square units.
- 3.MD.8. Measure areas by tiling with unit squares (square centimeters, square meters, square inches, square feet, and improvised units).
- 3.MD.9. Relate area to the operations of multiplication and addition.
- a. Find the area of a rectangle with wholenumber side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. For example, after tiling rectangles, develop a rule for finding the area of any rectangle. b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular

areas in mathematical reasoning.

- 4.MD.7. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand the following concepts of angle measurement:
- a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle
- 4.MD.8. Measure and draw angles in whole-number degrees using a protractor. Estimate and sketch angles of specified measure.

measure of n degrees.

4.MD.9. Recognize angle measure as additive. When an angle is divided into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems (e.g., by using an equation with a symbol for the unknown angle measure).

# Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

- 5.MD.5. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
- a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
- b. A solid figure that can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
- 5.MD.6. Estimate and measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and non-standard units.
- 5.MD.7. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
- a. Estimate and find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base.

  Demonstrate the associative property of multiplication by using the product of

Grade 3	Grade 4	Grade 5
c. Use area models (rectangular arrays) to represent the distributive property in mathematical reasoning. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths $a$ and $b + c$ is the sum of $a \times b$ and $a \times c$ .		b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems.
d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. For example, the area of a 7 by 8 rectangle can be determined by decomposing it into a 7 by 3 rectangle and a 7 by 5 rectangle.		c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.
Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.		
<ul> <li>3.MD.10. Solve real world and mathematical problems involving perimeters of polygons, including:</li> <li>finding the perimeter given the side lengths,</li> </ul>		
<ul> <li>finding an unknown side length,</li> <li>exhibiting rectangles with the same perimeter and different areas,</li> <li>exhibiting rectangles with the same area and different perimeters.</li> </ul>		

Grade 3	Grade 4	Grade 5
Geometry 3.G  Reason with shapes and their attributes.  3.G.1. Categorize shapes by different attribute classifications and recognize that shared attributes can define a larger category. Generalize to create examples or non-examples.  3.G.2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For	Geometry 4.G  Draw and identify lines and angles, and classify shapes by properties of their lines and angles.  4.G.1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular, parallel, and intersecting line segments. Identify these in two-dimensional (plane) figures.  4.G.2. Classify two-dimensional (plane) figures based on the presence or absence	Geometry 5.G  Graph points on the coordinate plane to solve real-world and mathematical problems.  5.G.1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates.  Understand that the first number indicates
example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.	of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.  4.G.3. Recognize a line of symmetry for a two-dimensional (plane) figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).  5.G.2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.  Classify two-dimensional (plane) figures into categories based on their properties.  5.G.3. Understand that attributes belonging to a category of two-

Grade 3	Grade 4	Grade 5
		dimensional (plane) figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all
		squares have four right angles.  5.G.4. Classify two-dimensional (plane) figures in a hierarchy based on attributes and properties.

### **Instructional Focus: Sixth Grade through Eighth Grade**

Grade 7

# In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing,

interpreting, and using expressions and

understanding of statistical thinking.

equations; and (4) developing

Grade 6

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multistep problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit

**Grade 8** 

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions (y/x = m or y = mx) as special linear equations (y = mx + b), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount  $m \cdot A$ . Students also use a linear equation to describe the association between two quantities in

- (2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.
- (3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining

### Grade 7

rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

- (2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.
- (3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in

### **Grade 8**

bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and yintercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial

the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as 3x = y) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected. Students in Grade 6 also build on their work with

### Grade 7

Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

### Grade 8

representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Grade 6	Grade 7	Grade 8
area in elementary school by reasoning		
about relationships among shapes to		
determine area, surface area, and		
volume. They find areas of right triangles,		
other triangles, and special quadrilaterals		
by decomposing these shapes,		
rearranging or removing pieces, and		
relating the shapes to rectangles. Using		
these methods, students discuss,		
develop, and justify formulas for areas of		
triangles and parallelograms. Students		
find areas of polygons and surface areas		
of prisms and pyramids by decomposing		
them into pieces whose area they can		
determine. They reason about right		
rectangular prisms with fractional side		
lengths to extend formulas for the		
volume of a right rectangular prism to		
fractional side lengths. They prepare for		
work on scale drawings and constructions		
in Grade 7 by drawing polygons in the		
coordinate plane.		

### **Alaska Mathematics Standards Grades 6-8**

Grade 6	Grade 7	Grade 8
Ratios and Proportional Relationships 6.RP	Ratios and Proportional Relationships 7.RP	
Understand ratio concepts and use ratio reasoning to solve problems.  6.RP.1. Write and describe the relationship in real life context between two quantities using ratio language. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."  6.RP.2. Understand the concept of a unit rate (a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship) and apply it to solve real world problems (e.g., unit pricing, constant speed).  For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger."	Analyze proportional relationships and use them to solve real-world and mathematical problems.  7.RP.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour or apply a given scale factor to find missing dimensions of similar figures.  7.RP.2. Recognize and represent proportional relationships between quantities. Make basic inferences or logical predictions from proportional relationships.  a. Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin).  b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships in real world situations.	

Grade 6	Grade 7	Grade 8
6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations).  a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios, and understand equivalencies.  b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?  c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.  d. Use ratio reasoning to convert measurement units between given measurement systems (e.g., convert kilometers to miles); manipulate and transform units appropriately when multiplying or dividing quantities.	c. Represent proportional relationships by equations and multiple representations such as tables, graphs, diagrams, sequences, and contextual situations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.  d. Understand the concept of unit rate and show it on a coordinate plane. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.  7.RP.3. Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.	

Grade 6	Grade 7	Grade 8
The Number System 6.NS	The Number System 7.NS	The Number System 8.NS
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	Know that there are numbers that are not rational, and approximate them by rational numbers.
6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions (e.g., by using visual fraction models and equations to represent the problem). For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?	7.NS.1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.  a. Show that a number and its opposite have a sum of 0 (additive inverses).  Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.  b. Understand addition of rational numbers  (p + q as the number located a distance  q  from p, in the positive or negative	8.NS.1. Classify real numbers as either rational (the ratio of two integers, a terminating decimal number, or a repeating decimal number) or irrational.  8.NS.2. Order real numbers, using approximations of irrational numbers, locating them on a number line. For example, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.  8.NS.3. Identify or write the prime factorization of a number using exponents. (L)
Compute fluently with multi-digit numbers and find common factors and multiples.	direction depending on whether <i>q</i> is positive or negative). Interpret sums of rational numbers by describing real-world contexts.	
6.NS.2. Fluently multiply and divide multi-digit whole numbers using the standard algorithm. Express the remainder as a whole number, decimal, or simplified fraction; explain or justify your	c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$ . Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.	

Grade 6	Grade 7	Grade 8
choice based on the context of the	d. Apply properties of operations as	
problem.	strategies to add and subtract rational	
	numbers.	
6.NS.3. Fluently add, subtract, multiply,		
and divide multi-digit decimals using the	7.NS.2. Apply and extend previous	
standard algorithm for each operation.	understandings of multiplication and	
Express the remainder as a terminating	division and of fractions to multiply and	
decimal, or a repeating decimal, or	divide rational numbers and use	
rounded to a designated place value.	equivalent representations.	
	a. Understand that multiplication is	
6.NS.4. Find the greatest common factor	extended from fractions to rational	
of two whole numbers less than or equal	numbers by requiring that operations	
to 100 and the least common multiple of	continue to satisfy the properties of	
two whole numbers less than or equal to	operations, particularly the distributive	
12. Use the distributive property to	property, leading to products such as (-	
express a sum of two whole numbers 1–	1)(-1) = 1 and the rules for multiplying	
100 with a common factor as a multiple	signed numbers. Interpret products of	
of a sum of two whole numbers with no	rational numbers by describing real-world	
common factor. For example, express 36	contexts.	
+ 8 as 4 (9 + 2).	b. Understand that integers can be divided,	
	provided that the divisor is not zero, and	
Apply and extend previous	every quotient of integers (with non-zero	
understandings of numbers to the	divisor) is a rational number. If $p$ and $q$ are	
system of rational numbers.	integers, then $-(p/q) = (-p)/q = p/(-q)$ .	
( NS 5 II a denote and the stance it is seen a	Interpret quotients of rational numbers by	
6.NS.5 Understand that positive and	describing real-world contexts.	
negative numbers describe quantities	c. Apply and name properties of	
having opposite directions or values (e.g.,	operations used as strategies to multiply	
temperature above/below zero, elevation	and divide rational numbers.	
above/below sea level, credits/debits,	d. Convert a rational number to a decimal	
positive/negative electric charge); use	using long division; know that the decimal	
positive and negative numbers to	form of a rational number terminates in 0s	
represent quantities in real-world	or eventually repeats.	
contexts, explain the meaning of 0 in each	e. Convert between equivalent fractions,	
situation.	decimals, or percents.	

Grade 6	Grade 7	Grade 8
6.NS.6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.  a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; Recognize that the opposite of the opposite of a number is the number itself [e.g., -(-3) = 3] and that 0 is its own opposite.  b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.  c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.  6.NS.7. Understand ordering and absolute value of rational numbers.  a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.	7.NS.3. Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)  For example, use models, explanations, number lines, real life situations, describing or illustrating the effect of arithmetic operations on rational numbers (fractions, decimals).	

Grade 7	Grade 8
	Grade 7

### Grade 6 Grade 7 **Grade 8** Expressions and Equations 6.EE Expressions and Equations 7.EE Expressions and Equations 8.EE Apply and extend previous Use properties of operations to generate Work with radicals and integer understandings of arithmetic to equivalent expressions. exponents. algebraic expressions. 7.EE.1. Apply properties of operations as 8.EE.1. Apply the properties (product, strategies to add, subtract, factor, expand quotient, power, zero, negative exponents, 6.EE.1. Write and evaluate numerical expressions involving whole-number and simplify linear expressions with and rational exponents) of integer rational coefficients. exponents. For example, multiply by exponents to generate equivalent powers of 10 and products of numbers numerical expressions. For example, $3^2 \times$ $3^{-5} = 3^{-3} = 1/3^3 = 1/27$ 7.EE.2. Understand that rewriting an using exponents. $(7^3 = 7 \cdot 7 \cdot 7)$ expression in different forms in a problem 6.EE.2. Write, read, and evaluate context can shed light on the problem and 8.EE.2. Use square root and cube root expressions in which letters stand for how the quantities in it are related. For symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$ , numbers. example, a + 0.05a = 1.05a means that "increase by 5%" is the same as where p is a positive rational number. a. Write expressions that record operations with numbers and with letters Evaluate square roots of small perfect "multiply by 1.05." standing for numbers. For example. squares and cube roots of small perfect Solve real-life and mathematical express the calculation "Subtract y from cubes. Know that $\sqrt{2}$ is irrational. 5" as 5 - y. problems using numerical and algebraic expressions and equations. 8.EE.3. Use numbers expressed in the b. Identify parts of an expression using form of a single digit times an integer mathematical terms (sum, term, product, 7.EE.3. Solve multi-step real-life and power of 10 to estimate very large or very factor, quotient, coefficient); view one or mathematical problems posed with small quantities, and to express how many more parts of an expression as a single positive and negative rational numbers in times as much one is than the other. For entity. For example, describe the any form and assess the reasonableness of example, estimate the population of the expression answers using mental computation and *United States as* $3 \times 10^8$ *and the* 2(8+7) as a product of two factors: estimation strategies. For example: If a population of the world as $7 \times 10^9$ , and view (8 + 7) as both a single entity and a woman making \$25 an hour gets a 10% determine that the world population is sum of two terms. raise, she will make an additional 1/10 of c. Evaluate expressions and formulas. more than 20 times larger. her salary an hour, or \$2.50, for a new Include formulas used in real-world salary of \$27.50. If you want to place a 8.EE.4. Perform operations with numbers problems. Perform arithmetic operations, towel bar 9 3/4 inches long in the center expressed in scientific notation, including including those involving whole number of a door that is 27 1/2 inches wide, you problems where both standard notation

will need to place the bar about 9 inches

and scientific notation are used. Use

scientific notation and choose units of

exponents, in the conventional order with

or without parentheses. (Order of Operations)

6.EE.3. Apply the properties of operations to generate equivalent expressions. Model (e.g., manipulatives, graph paper) and apply the distributive, commutative, identity, and inverse properties with integers and variables by writing equivalent expressions. For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x.

6.EE.4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.

# Reason about and solve one-variable equations and inequalities.

6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. For example: does 5 make 3x > 7 true?

### Grade 7

from each edge; this estimate can be used as a check on the exact computation.

7.EE.4. Use variables to represent quantities in a real-world or mathematical problem, and construct multi-step equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form px + q = r and p(x + q)q = r, where p, q, and r are specific rational numbers. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? b. Solve word problems leading to inequalities of the form px + q > r or px + q > rq < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.

### Grade 8

appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology.

# Understand the connections between proportional relationships, lines, and linear equations.

8.EE.5. Graph linear equations such as y = mx + b, interpreting m as the slope or rate of change of the graph and b as the y-intercept or starting value. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

8.EE.6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

# Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.7. Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show

Grade 6	Grade 7	Grade 8
6.EE.6. Use variables to represent		which of these possibilities is the case by
numbers and write expressions when		successively transforming the given
solving a real-world or mathematical		equation into simpler forms, until an
problem; understand that a variable can		equivalent equation of the form $x = a$ , $a = a$
represent an unknown number, or,		a, or $a = b$ results (where $a$ and $b$ are
depending on the purpose at hand, any		different numbers).
number in a specified set.		b. Solve linear equations with rational
_		coefficients, including equations whose
6.EE.7. Solve real-world and		solutions require expanding expressions
mathematical problems by writing and		using the distributive property and
solving equations of the form $x + p = q$		combining like terms.
and $px = q$ for cases in which $p$ , $q$ and $x$		
are all nonnegative rational numbers.		8.EE.8. Analyze and solve systems of
		linear equations.
6.EE.8. Write an inequality of the form $x$		a. Show that the solution to a system of
> c or $x < c$ to represent a constraint or		two linear equations in two variables is the
condition in a real-world or mathematical		intersection of the graphs of those
problem. Recognize that inequalities of		equations because points of intersection
the form $x > c$ or $x < c$ have infinitely		satisfy both equations simultaneously.
many solutions; represent solutions of		b. Solve systems of two linear equations in
such inequalities on number line		two variables and estimate solutions by
diagrams.		graphing the equations. Simple cases may
		be done by inspection. For example, $3x +$
Represent and analyze quantitative		2y = 5 and $3x + 2y = 6$ have no solution
relationships between dependent and		because $3x + 2y$ cannot simultaneously be
independent variables.		5 and 6.
6.EE.9. Use variables to represent two		c. Solve real-world and mathematical
quantities in a real-world problem that		problems leading to two linear equations
change in relationship to one another;		in two variables. For example, given
write an equation to express one quantity,		coordinates for two pairs of points,
thought of as the dependent variable, in		determine whether the line through the
terms of the other quantity, thought of as		first pair of points intersects the line
the independent variable. Analyze the		through the second pair.
relationship between the dependent and		
independent variables using graphs and		

Grade 6	Grade 7	Grade 8
tables, and relate these to the equation.		
For example, in a problem involving		
motion at constant speed, list and graph		
ordered pairs of distances and times, and		
write the equation $d = 65t$ to represent		
the relationship between distance and		
time.		

Grade 6	Grade 7	Grade 8
Geometry 6.G	Geometry 7.G	Geometry 8.G
Solve real-world and mathematical problems involving area, surface area, and volume.	Draw, construct, and describe geometrical figures and describe the relationships between them.	Understand congruence and similarity using physical models, transparencies, or geometry software.
6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing or decomposing into other polygons (e.g., rectangles and triangles). Apply these techniques in the context of solving real-world and mathematical problems.	7.G.1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.  7.G.2. Draw (freehand, with ruler and protractor, and with technology)	8.G.1. Through experimentation, verify the properties of rotations, reflections, and translations (transformations) to figures on a coordinate plane). a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same
6.G.2. Apply the standard formulas to find volumes of prisms. Use the attributes and properties (including shapes of bases)	geometric shapes including polygons and circles with given conditions. Focus on constructing triangles from three measures	measure. c. Parallel lines are taken to parallel lines.
of prisms to identify, compare or describe three-dimensional figures including prisms and cylinders.	of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	8.G.2. Demonstrate understanding of congruence by applying a sequence of translations, reflections, and rotations on two-dimensional figures. Given two
6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; determine the length of a side joining the coordinates of vertices with the same first	7.G.3. Describe the two-dimensional figures, i.e., cross-section, that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms	congruent figures, describe a sequence that exhibits the congruence between them.
or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	and right rectangular pyramids.  Solve real-life and mathematical problems involving angle measure,	8.G.3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
6.G.4. Represent three-dimensional figures (e.g., prisms) using nets made up of rectangles and triangles, and use the nets to find the surface area of these	7.G.4. Know the formulas for the area and circumference of a circle and use them to	8.G.4. Demonstrate understanding of similarity, by applying a sequence of translations, reflections, rotations, and
figures. Apply these techniques in the context of solving real-world and mathematical problems.	solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	dilations on two-dimensional figures.  Describe a sequence that exhibits the similarity between them.

Grade 6	Grade 7	Grade 8
6.G.5. Identify, compare or describe attributes and properties of circles (radius, and diameter). (L)	7.G.5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.  7.G.6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	<ul> <li>8.G.5. Justify using informal arguments to establish facts about</li> <li>the angle sum of triangles (sum of the interior angles of a triangle is 180°),</li> <li>measures of exterior angles of triangles,</li> <li>angles created when parallel lines are cut be a transversal (e.g., alternate interior angles), and</li> <li>angle-angle criterion for similarity of triangles.</li> <li>Understand and apply the Pythagorean Theorem.</li> <li>8.G.6. Explain the Pythagorean Theorem and its converse.</li> <li>8.G.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</li> <li>8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</li> <li>Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.</li> <li>8.G.9. Identify and apply the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.</li> </ul>

Grade 6	Grade 7	Grade 8
Statistics and Probability 6.SP	Statistics and Probability 7.SP	Statistics and Probability 8.SP
Develop understanding of statistical variability.	Use random sampling to draw inferences about a population.	Investigate patterns of association in bivariate data.
6.SP.1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.  6.SP.2. Understand that a set of data has a distribution that can be described by its center (mean, median, or mode), spread (range), and overall shape and can be used to answer a statistical question.  6.SP.3. Recognize that a measure of center (mean, median, or mode) for a numerical data set summarizes all of its values with a single number, while a measure of variation (range) describes how its values vary with a single number.  Summarize and describe distributions.	7.SP.1. Understand that statistics can be used to gain information about a population by examining a reasonably sized sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.  7.SP.2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.  Draw informal comparative inferences	8.SP.1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.  8.SP.2. Explain why straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.  8.SP.3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and <i>y</i> -intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5
6.SP.4. Display numerical data in plots on	about two populations.	cm in mature plant height.
a number line, including dot or line plots, histograms and box (box and whisker)	7 SD 3. Informally assess the degree of	& SD A. Construct and interpret a two way
plots.	7.SP.3. Informally assess the degree of visual overlap of two numerical data	8.SP.4. Construct and interpret a two-way table summarizing data on two categorical
	distributions with similar variabilities,	variables collected from the same subjects

- 6.SP.5. Summarize numerical data sets in relation to their context, such as by:
- a. Reporting the number of observations (occurrences).
- b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
- c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range), as well as describing any overall pattern and any outliers with reference to the context in which the data were gathered.
- d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
- 6.SP.6. Analyze whether a game is mathematically fair or unfair by explaining the probability of all possible outcomes. (L)
- 6.SP.7. Solve or identify solutions to problems involving possible combinations (e.g., if ice cream sundaes come in 3 flavors with 2 possible toppings, how many different sundaes can be made using only one flavor of ice cream with one topping?) (L)

### Grade 7

measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.

7.SP.4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

# Investigate chance processes and develop, use, and evaluate probability models.

7.SP.5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

### Grade 8

and use relative frequencies to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

### Functions 8.F

# Define, evaluate, and compare functions.

- 8.F.1. Understand that a function is a rule that assigns to each input (the domain) exactly one output (the range). The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. For example, use the vertical line test to determine functions and non-functions.
- 8.F.2. Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
- 8.F.3. Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions

Grade 6	Grade 7	Grade 8
	7.SP.6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.  7.SP.7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.  a. Design a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.  b. Design a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?	that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points $(1,1)$ , $(2,4)$ and $(3,9)$ , which are not on a straight line.  Use functions to model relationships between quantities.  8.F.4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two $(x, y)$ values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.  8.F.5. Given a verbal description between two quantities, sketch a graph. Conversely, given a graph, describe a possible real-world example. For example, graph the position of an accelerating car or tossing a ball in the air.

Grade 6	Grade 7	Grade 8
	7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?	

## **High School Mathematical Content Standards**

### **Courses and Transitions**

The high school standards specify the mathematics that all students should study in order to be career and college ready. They are organized into conceptual categories, which are intended to portray a coherent view of high school mathematics. A student's work with any set of standards crosses a number of traditional course boundaries. For example, the Functions Standards would apply to different courses such as Algebra I or Algebra II.

These standards do not mandate the sequence of high school courses. However, the organization of high school courses is a critical component to implementation of the standards. It is a district decision how to design course offerings covering the mathematics standards. Districts can use the traditional approach of Algebra I, Geometry, and Algebra II or implement an integrated approach. There are various high school math pathways to be considered and likely additional model pathways based on these standards will become available as well.

The standards themselves do not dictate curriculum, pedagogy, or delivery of content. In particular, districts may handle the transition to high school in different ways. For example, many students in the U.S. today take Algebra I in the 8th grade, and in some districts and states this is a requirement. By completing grade 7 standards successfully, students have met the prerequisites and are prepared for Algebra I by 8th grade. The standards are designed to permit districts and states to continue existing policies concerning Algebra I in 8th grade.

Another major transition is the transition from high school to post-secondary education for college and careers. The evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume. Because important standards for college and career readiness are distributed across grades and courses, systems for evaluating college and career readiness should reach as far back in the standards as grades 6-8.

### Narrative of Standards - Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

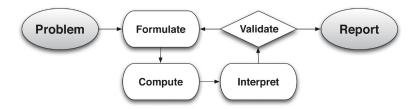
The basic modeling cycle is summarized in the diagram below. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

**Modeling Standards**. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by an asterisk (\*).



### Narrative of Standards - Number and Quantity

**Numbers and Number Systems.** During the years from kindergarten to 8th grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3... . Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that  $(5^{1/3})^3$  should be  $5^{(1/3)3} = 5^1 = 5$  and that  $5^{1/3}$  should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

**Quantities.** In real-world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

### **Number and Quantity Standards**

The Real Number System N – RN

### Extend the properties of exponents to rational exponents.

- N-RN.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define  $5^{1/3}$  to be the cube root of 5 because we want  $(5^{1/3})^3 = 5(1/3)^3$  to hold, so  $(5^{1/3})^3$  must equal 5.
- N-RN.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. *For example: Write equivalent representations that utilize both positive and negative exponents.*

### Use properties of rational and irrational numbers.

N-RN.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities\* N – Q

### Reason quantitatively and use units to solve problems.

- N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.
- N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

### **The Complex Number System**

N - CN

### Perform arithmetic operations with complex numbers.

- N-CN.1. Know there is a complex number i such that  $i^2 = -1$ , and every complex number has the form a + bi with a and b real.
- N-CN.2. Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- N-CN.3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

### Represent complex numbers and their operations on the complex plane.

- N-CN.4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
- N-CN.5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example,  $(1 \sqrt{3}i)^3 = 8$  because  $(1 \sqrt{3}i)$  has modulus 2 and argument 120°.
- N-CN.6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

### Use complex numbers in polynomial identities and equations.

- N-CN.7. Solve quadratic equations with real coefficients that have complex solutions.
- N-CN.8. (+) Extend polynomial identities to the complex numbers. For example, rewrite  $x^2 + 4$  as (x + 2i)(x 2i).
- N-CN.9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

### **Vector and Matrix Quantities**

N - VM

### Represent and model with vector quantities.

- N-VM.1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g.,  $\mathbf{v}$ ,  $|\mathbf{v}|$ ,  $|\mathbf{v}|$ ,  $|\mathbf{v}|$ ,  $|\mathbf{v}|$ ).
- N-VM.2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- N-VM.3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

### Perform operations on vectors.

- N-VM.4. (+) Add and subtract vectors.
  - a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
  - b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
  - c. Understand vector subtraction  $\mathbf{v} \mathbf{w}$  as  $\mathbf{v} + (-\mathbf{w})$ , where  $-\mathbf{w}$  is the additive inverse of  $\mathbf{w}$ , with the same magnitude as  $\mathbf{w}$  and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

- N-VM.5. (+) Multiply a vector by a scalar.
  - a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as  $c(v_x, v_y) = (cv_x, cv_y)$ .
  - b. Compute the magnitude of a scalar multiple  $c\mathbf{v}$  using  $||c\mathbf{v}|| = |c|\mathbf{v}$ . Compute the direction of  $c\mathbf{v}$  knowing that when  $|c|\mathbf{v} \neq 0$ , the direction of  $c\mathbf{v}$  is either along  $\mathbf{v}$  (for c > 0) or against  $\mathbf{v}$  (for c < 0).

### Perform operations on matrices and use matrices in applications.

- N-VM.6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
- N-VM.7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
- N-VM.8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
- N-VM.9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
- N-VM.10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
- N-VM.11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
- N-VM.12. (+) Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.

### Narrative of Standards - Algebra

**Expressions.** An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

**Equations and Inequalities.** An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of  $x^2 - 2 = 0$  are real numbers, not rational numbers; and the solutions of  $x^2 + 2 = 0$  are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid,  $A = ((b_1+b_2)/2)h$ , can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

### **Algebra Standards**

### **Seeing Structure in Expressions**

A - SSE

### Interpret the structure of expressions.

A-SSE.1. Interpret expressions that represent a quantity in terms of its context.\*

- a. Interpret parts of an expression, such as terms, factors, and coefficients.
- b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret  $P(1+r)^n$  as the product of P and a factor not depending on P.
- A-SSE.2. Use the structure of an expression to identify ways to rewrite it. For example, see  $x^4 y^4$  as  $(x^2)^2 (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 y^2)(x^2 + y^2)$ .

### Write expressions in equivalent forms to solve problems.

- A-SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.\*
  - a. Factor a quadratic expression to reveal the zeros of the function it defines. For example,  $x^2 + 4x + 3 = (x + 3)(x + 1)$ .
  - b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. For example,  $x^2 + 4x + 3 = (x + 2)^2 1$ .
  - c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15<sup>t</sup> can be rewritten as  $(1.15^{1/12})^{12t} \approx 1.012^{12t}$  to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
- A-SSE.4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.\*

### **Arithmetic with Polynomials and Rational Expressions**

A - APR

### Perform arithmetic operations on polynomials.

A-APR.1. Add, subtract, and multiply polynomials. Understand that polynomials form a system similar to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.

### Understand the relationship between zeros and factors of polynomials.

A-APR.2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x - a is p(a), so p(a) = 0

if and only if (x - a) is a factor of p(x).

A-APR.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

### Use polynomial identities to solve problems.

A-APR.4. Prove polynomial identities and use them to describe numerical relationships.

For example, the polynomial identity  $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$  can be used to generate Pythagorean triples.

A-APR.5. (+) Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

## Rewrite rational expressions.

- A-APR.6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
- A-APR.7. (+) Add, subtract, multiply, and divide rational expressions. Understand that rational expressions form a system similar to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

## **Creating Equations and Inequalities\***

A - CED

## Create equations and inequalities that describe numbers or relationships.

- A-CED.1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
- A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- A-CED.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing cost constraints in various situations.
- A-CED.4. Rearrange formulas (literal equations) to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

## **Reasoning with Equations and Inequalities**

A – REI

## Understand solving equations as a process of reasoning and explain the reasoning.

A-REI.1. Apply properties of mathematics to justify steps in solving equations in one variable.

A-REI.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

#### Solve equations and inequalities in one variable.

A-REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A-REI.4. Solve quadratic equations in one variable.

- a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form  $(x-p)^2=q$  that has the same solutions. Derive the quadratic formula from this form.
- b. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers a and b.

#### Solve systems of equations.

- A-REI.5. Show that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- A-REI.6. Solve systems of linear equations exactly and approximately, e.g., with graphs or algebraically, focusing on pairs of linear equations in two variables.
- A-REI.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = -3x and the circle  $x^2 + y^2 = 3$ .
- A-REI.8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
- A-REI.9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension
  - $3 \times 3$  or greater).

## Represent and solve equations and inequalities graphically.

- A-REI.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- A-REI.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.\*
- A-REI.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

## **Narrative of Standards - Functions**

**Functions.** describe situations where one quantity determines another. For example, the return on \$10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule T(v) = 100/v expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like f(x) = a + bx; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates. Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

## **Functions Standards**

Interpreting Functions F - IF

## Understand the concept of a function and use function notation.

- F-IF.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- F-IF.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- F-IF.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for  $n \ge 1$ .

### Interpret functions that arise in applications in terms of the context.

- F-IF.4. For a function that models a relationship between two quantities,
  - interpret key features of graphs and tables in terms of the quantities, and
  - sketch graphs showing key features given a verbal description of the relationship.

    Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.\*
- F-IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then negative numbers would be an inappropriate domain for the function.\*
- F-IF.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.\*

#### Analyze functions using different representations.

- F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*
  - a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
  - b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
  - c. Graph polynomial functions, identifying zeros (using technology) or algebraic methods when suitable factorizations are available, and showing end behavior.
  - d. (+) Graph rational functions, identifying zeros and discontinuities (asymptotes/holes) using technology, and algebraic methods when suitable factorizations are available, and showing end behavior.
  - e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- F-IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
  - a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
  - b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as  $y = (1.02)^t$ ,  $y = (0.97)^t$ ,  $y = (1.01)^{12t}$ ,  $y = (1.2)^{t/10}$ , and classify them as representing exponential growth or decay.
- F-IF.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically, in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions F - BF

## Build a function that models a relationship between two quantities.

- F-BF.1. Write a function that describes a relationship between two quantities.\*
  - a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
  - b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
  - c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
- F-BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.\*

## Build new functions from existing functions.

- F-BF.3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- F-BF.4. Find inverse functions.
  - a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example,  $f(x) = 2x^3$  for x > 0 or f(x) = (x + 1)/(x - 1) for  $x \ne 1$ .
  - b. (+) Verify by composition that one function is the inverse of another.
  - c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
  - d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

F-BF.5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

#### Linear, Quadratic, and Exponential Models\*

F - LE

#### Construct and compare linear, quadratic, and exponential models and solve problems.

- F-LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
  - a. Show that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
  - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
  - c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- F-LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or input-output table of values.
- F-LE.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- F-LE.4. For exponential models, express as a logarithm the solution to  $ab^{ct} = d$  where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

## Interpret expressions for functions in terms of the situation they model.

F-LE.5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F - TF

## Extend the domain of trigonometric functions using the unit circle.

- F-TF.1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- F-TF.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- F-TF.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosines, and tangent for  $\pi-x$ ,  $\pi+x$ , and  $2\pi-x$  in terms of their values for x, where x is any real number.
- F-TF.4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

#### Model periodic phenomena with trigonometric functions.

- F-TF.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.\*
- F-TF.6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- F-TF.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.\*

## Prove and apply trigonometric identities.

- F-TF.8. Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to calculate trigonometric ratios.
- F-TF.9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

## Narrative of Standards - Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to nonright triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

**Connections to Equations.** The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

## **Geometry Standards**

Congruence G - CO

## Experiment with transformations in the plane.

- G-CO.1. Demonstrates understanding of key geometrical definitions, including angle, circle, perpendicular line, parallel line, line segment, and transformations in Euclidian geometry. Understand undefined notions of point, line, distance along a line, and distance around a circular arc.
- G-CO.2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- G-CO.3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
- G-CO.4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
- G-CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

## Understand congruence in terms of rigid motions.

- G-CO.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- G-CO.7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- G-CO.8. Explain how the criteria for triangle congruence (ASA, SAS, SSS, AAS, and HL) follow from the definition of congruence in terms of rigid motions.

#### Prove geometric theorems.

- G-CO.9. Using methods of proof including direct, indirect, and counter examples to prove theorems about lines and angles.

  Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
- G-CO.10. Using methods of proof including direct, indirect, and counter examples to prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- G-CO.11. Using methods of proof including direct, indirect, and counter examples to prove theorems about parallelograms.

  Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

#### Make geometric constructions.

- G-CO.12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
- G-CO.13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

## Understand similarity in terms of similarity transformations.

- G-SRT.1. Verify experimentally the properties of dilations given by a center and a scale factor:
  - a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
  - b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
- G-SRT.2. Given two figures, use the definition of similarity in terms of transformations to explain whether or not they are similar.
- G-SRT.3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

## Prove theorems involving similarity.

- G-SRT.4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely.*
- G-SRT.5. Apply congruence and similarity properties and prove relationships involving triangles and other geometric figures.

## Define trigonometric ratios and solve problems involving right triangles.

- G-SRT.6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- G-SRT.7. Explain and use the relationship between the sine and cosine of complementary angles.
- G-SRT.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.\*

## Apply trigonometry to general triangles.

- G-SRT.9. (+) Derive the formula A = 1/2  $ab \sin(C)$  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- G-SRT.10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
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G-SRT.11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Circles G - C

## Understand and apply theorems about circles.

- G-C.1. Prove that all circles are similar.
- G-C.2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
- G-C.3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
- G-C.4. (+) Construct a tangent line from a point outside a given circle to the circle.

## Find arc lengths and areas of sectors of circles.

G-C.5. Use and apply the concepts of arc length and areas of sectors of circles. Determine or derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

## **Expressing Geometric Properties with Equations**

G - GPE

## Translate between the geometric description and the equation for a conic section.

- G-GPE.1. Determine or derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
- G-GPE.2. Determine or derive the equation of a parabola given a focus and directrix.
- G-GPE.3. (+) Derive the equations of ellipses and hyperbolas given foci and directrices.

#### Use coordinates to prove simple geometric theorems algebraically.

- G-GPE.4. Perform simple coordinate proofs. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point  $(1, \sqrt{3})$  lies on the circle centered at the origin and containing the point (0, 2).
- G-GPE.5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
- G-GPE.6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
- G-GPE.7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.\*

#### **Geometric Measurement and Dimension**

G-GMD

#### Explain volume formulas and use them to solve problems.

- G-GMD.1. Explain how to find the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- G-GMD.2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
- G-GMD.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. *For example: Solve problems requiring determination of a dimension not given.*\*

## Visualize relationships between two-dimensional and three-dimensional objects.

G-GMD.4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry G - MG

## Apply geometric concepts in modeling situations.

G-MG.1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).\*

- G-MG.2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).\*
- G-MG.3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).\*

## Narrative of Standards - Statistics and Probability\*

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

**Connections to Functions and Modeling**. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

## **Statistics and Probability Standards\***

## **Interpreting Categorical and Quantitative Data**

S - ID

#### Summarize, represent, and interpret data on a single count or measurement variable.

- S-ID.1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
- S-ID.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
- S-ID.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). For example: Justify why median price of homes or income is used instead of the mean.
- S-ID.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages.

  Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

## Summarize, represent, and interpret data on two categorical and quantitative variables.

- S-ID.5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
- S-ID.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
  - a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*

- b. Informally assess the fit of a function by plotting and analyzing residuals. For example: Describe solutions to problems that require interpolation and extrapolation.
- c. Fit a linear function for a scatter plot that suggests a linear association.

## Interpret linear models.

- S-ID.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- S-ID.8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
- S-ID.9. Distinguish between correlation and causation.

## **Making Inferences and Justifying Conclusions**

S-IC

### Understand and evaluate random processes underlying statistical experiments.

- S-IC.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
- S-IC.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

## Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

- S-IC.3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
- S-IC.4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
- S-IC.5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
- S-IC.6. Evaluate reports based on data.
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## Understand independence and conditional probability and use them to interpret data.

- S-CP.1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
- S-CP.2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- S-CP.3. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
- S-CP.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in 10th grade. Do the same for other subjects and compare the results.
- S-CP.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

## Use the rules of probability to compute probabilities of compound events in a uniform probability model.

- S-CP.6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.
- S-CP.7. Apply the Addition Rule, P(A or B) = P(A) + P(B) P(A and B), and interpret the answer in terms of the model.
- S-CP.8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
- S-CP.9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

## Calculate expected values and use them to solve problems.

- S-MD.1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
- S-MD.2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
- S-MD.3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.
- S-MD.4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?

## Use probability to evaluate outcomes of decisions.

- S-MD.5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
  - a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.
  - b. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.
- S-MD.6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
- S-MD.7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

# **Glossary for Alaska Mathematics Standards**

## addition and subtraction within 5, 10, 20, 100, or 1000

Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 - 5 = 9 is a subtraction within 20, and 55 - 18 = 37 is a subtraction within 100.

#### additive inverses

Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and -3/4 are additive inverses of one another because 3/4 + (-3/4) = (-3/4) + 3/4 = 0.

## associative property of addition

See Table 3 in this Glossary.

## associative property of multiplication

See Table 3 in this Glossary.

#### bivariate data

Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team. Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.<sup>1</sup>

#### cardinality

Cardinal numbers, known as the "counting numbers," indicate quantity.

#### commutative property

See Table 3 in this Glossary.

## complex fraction

A fraction A/B where A and/or B are fractions (B nonzero).

#### computation algorithm

A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

#### computation strategy

Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

<sup>&</sup>lt;sup>1</sup> Adapted from Wisconsin Department of Public Instruction, please visit website: <u>Standards Math Glossary</u>, accessed Mar 2, 2010.

#### congruent

Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

#### counting on

A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

## dot plot

See: line plot

#### dilation

A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

## expanded form

A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

#### expected value

For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

#### first quartile

For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set  $\{1, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$ , the first quartile is  $6.^2$  See also: median, third quartile, interquartile range.

#### fraction

A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

#### identity property of 0

See Table 3 in this Glossary.

#### independently combined probability models

Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

<sup>&</sup>lt;sup>2</sup> Many different methods for computing quartiles are in use. The method defined here is sometimes called the More and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education*, Volume 14, number 3 (2006).

#### integer

A number expressible in the form a or -a for some whole number a.

#### interquartile range

A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set  $\{1, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$ , the interquartile range is 15 - 6 = 9. See also: first quartile, third quartile.

### line plot

A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.<sup>3</sup>

#### mean

A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.<sup>4</sup> Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

#### mean absolute deviation

A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

#### median

A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

#### midline

In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example:  $72 \div 8 = 9$ .

#### multiplicative inverses

Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because  $3/4 \times 4/3 = 4/3 \times 3/4 = 1$ .

#### number line diagram.

A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

#### ordinality

Ordinal numbers indicate the order or rank of things in a set (e.g., sixth in line; fourth place).

<sup>&</sup>lt;sup>3</sup> Adapted from Wisconsin Department of Public Instruction, op. cit.

<sup>&</sup>lt;sup>4</sup> To be more precise, this defines the *arithmetic mean*.

#### percent rate of change

A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

## probability distribution

The set of possible values of a random variable with a probability assigned to each.

## probability

A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

## probability model

A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

#### properties of equality

See Table 4 in this Glossary.

## properties of inequality

See Table 5 in this Glossary.

#### properties of operation

See Table 3 in this Glossary.

#### random variable

An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

#### rational expression

A quotient of two polynomials with a non-zero denominator.

#### rational number

A number expressible in the form a/b or -a/b for some fraction a/b. The rational numbers include the integers.

## rectilinear figure

A polygon all angles of which are right angles.

#### rigid motion

A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

#### repeating decimal

The decimal form of a rational number. See also: terminating decimal.

#### sample space

In a probability model for a random process, a list of the individual outcomes that are to be considered.

## scatter plot

A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.<sup>5</sup>

## similarity transformation

A rigid motion followed by a dilation.

#### tape diagram

A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

#### terminating decimal

A decimal is called terminating if its repeating digit is 0.

#### third quartile

For a data set with median M, the third quartile is the median of the data values greater than M. For example: For the data set  $\{2, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$ , the third quartile is 15. See also: median, first quartile, interquartile range.

## transitivity principle for indirect measurement

If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

#### uniform probability model

A probability model which assigns equal probability to all outcomes. See also: probability model.

#### vector

A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

#### visual fraction model

A tape diagram, number line diagram, or area model.

#### whole numbers

The numbers 0, 1, 2, 3,...

<sup>&</sup>lt;sup>5</sup> Adapted from Wisconsin Department of Public Instruction, op. cit.

Table 1: Common addition and subtraction situations<sup>1</sup>

	Result Unknown	Change Unknown	Start Unknown
Add To  Take from	Result Unknown  Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?  2 + 3 = ?  Five apples were on the table. I ate two apples. How many apples are on the table now? 5 - 2 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two?  2 + ? = 5  Five apples were on the table. I ate some apples. Then there were three apples. How many apples	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5 Some apples were on the table. I ate two apples. Then there were three apples. How many apples
		did I eat? 5 – ? = 3	were on the table before? ? – 2 = 3
	Total Unknown	Addend Unknown	Both Addends Unknown <sup>2</sup>
Put Together/ Take Apart <sup>3</sup>	Three red apples and two green apples are on the table. How many apples are on the table?  3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare <sup>4</sup>	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 - 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

<sup>&</sup>lt;sup>1</sup>Adapted from Box 2-4 of the National Research Council (2009, op. cit., pp. 32, 33).

<sup>&</sup>lt;sup>2</sup>These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

Table 2: Common multiplication and division situations<sup>5</sup>

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
Equal Groups	3 x 6 = ?  There are 3 bags with 6 plums in each bag. How many plums are there in all?	3 x? = 18, and 18 ÷ 3 = ?  If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?	? x 6 = 18, and 18 ÷ 6 = ?  If 18 plums are to be packed 6 to a bag, then how many bags are needed?
	Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	Measurement example. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Array <sup>6</sup> , Area <sup>7</sup>	There are 3 rows of apples with 6 apples in each row. How many apples are there?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?
	Area example. What is the area of a 3 cm by 6 cm rectangle?	Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?
	Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it	Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was	Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the

<sup>&</sup>lt;sup>3</sup>Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

<sup>&</sup>lt;sup>4</sup>For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	is stretched to be 3	at first. How long was the	rubber band now as it was
	times as long?	rubber band at first?	at first?
General	$a \times b = ?$	a x ? = p, and p ÷ a = ?	? $x b = p$ , and $p \div b = ?$

<sup>&</sup>lt;sup>5</sup>The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

## **Table 3: The properties of operations**

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	(a+b)+c=a+(b+c)
Commutative property of addition	a+b=b+a
Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every $a$ there exists $-a$ so that $a + (-a) = (-a) + a = 0$ .
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \ne 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$ .
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

<sup>&</sup>lt;sup>6</sup>The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

<sup>&</sup>lt;sup>7</sup>Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

## **Table 4: The properties of equality**

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a
Symmetric property of equality	If $a = b$ , then $b = a$ .
Transitive property of equality	If $a = b$ and $b = c$ , then $a = c$ .
Addition property of equality	If $a = b$ , then $a + c = b + c$ .
Subtraction property of equality	If $a = b$ , then $a - c = b - c$ .
Multiplication property of equality	If $a = b$ , then $a \times c = b \times c$ .
Division property of equality	If $a = b$ , then $b$ may be substituted for $a$ in any
	expression containing $a$ .
Substitution property of equality	If $a = b$ and $c \ne 0$ , then $a \div c = b \div c$ .

## **Table 5: The properties of inequality**

Here a, b and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$ , $a = b$ , $a > b$ .
If $a > b$ and $b > c$ then $a > c$ .
If $a > b$ , then $b < a$ .
If $a > b$ , then $-a < -b$ .
If $a > b$ , then $a \pm c > b \pm c$ .
If $a > b$ and $c > 0$ , then $a \times c > b \times c$ .
If $a > b$ and $c < 0$ , then $a \times c < b \times c$ .
If $a > b$ and $c > 0$ , then $a \div c > b \div c$ .
If $a > b$ and $c < 0$ , then $a \div c < b \div c$ .



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## **Publication Notice:**

The Department of Education & Early Development published this document. It was printed at a cost of \$3.53 per copy in Juneau and paid for with state funds. The department's intent is to inform Alaskans about education standards adopted by the State Board of Education & Early Development.

Information: 465-2900